Statistical Mechanics: Problems 11.1

1. **Problem:** Show that Bose-Einstein condensation is not possible 2-dimensions.
Solution: The total number of bosons in the gas, at a temperature T is given by

$$\langle N\rangle = \sum_k \langle n_k\rangle = \sum_k \frac{1}{z^{-1}e^{\beta\epsilon_k} - 1}$$

In 2 dimensions, we assume that the sum goes over to an integral over p_x, p_y:

$$\sum_k \rightarrow \frac{A}{h^2}\int_0^\infty 2\pi p dp$$

Using this, we get

$$\begin{aligned}
\langle N\rangle &= \langle n_0\rangle + \frac{2\pi A}{h^2}\int_0^\infty \frac{p}{z^{-1}e^{\beta p^2/2m} - 1} dp \\
&= \langle n_0\rangle + \frac{2\pi A}{h^2}\int_0^\infty \frac{pze^{-\beta p^2/2m}}{1 - ze^{-\beta p^2/2m}} dp \\
&= \langle n_0\rangle + \frac{2\pi A}{h^2}\frac{m}{\beta}\int_{1-z}^1 \frac{dt}{t} \qquad (t = 1 - ze^{-\beta p^2/2m}) \\
&= \langle n_0\rangle - \frac{A}{\lambda^2}\log(1-z)
\end{aligned}$$

The term $-\frac{A}{\lambda^2}\log(1-z)$ represents the number of particles in all the excited states. This term goes to infinity as $z \rightarrow 1$. Thus, the excited states have infinite capacity to hold particles, and the particles are never pushed to the ground state.
Hence, no Bose-Einstein condensation in 2-dimensions!