ATOMS OF SPACETIME

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Fourth Abdus Salaam Memorial Lecture (2005-2006)

WHAT WILL BE THE VIEW REGARDING GRAVITY AND SPACETIME IN THE YEAR 2206 ?

$$\text{CLASSICAL GRAVITY} = \begin{cases} \text{CONDENSED MATTER PHYSICS} \\ \text{OF SPACETIME SUBSTRUCTURE} \end{cases}$$

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HOW GRAVITY RESPONDS TO AND MODIFIES THE STRUCTURE

OF THE VACUUM IS PROBABLY THE KEY QUESTION

IN THEORETICAL PHYSICS TODAY.

A WORD FROM THE SPONSORS

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”Of general theory of relativity, you will be convinced once you have studied it. Therefore, I am not going to defend it with a single word. ”

A.Einstein [in a letter to A.Sommerfeld, 6 Feb 1916]

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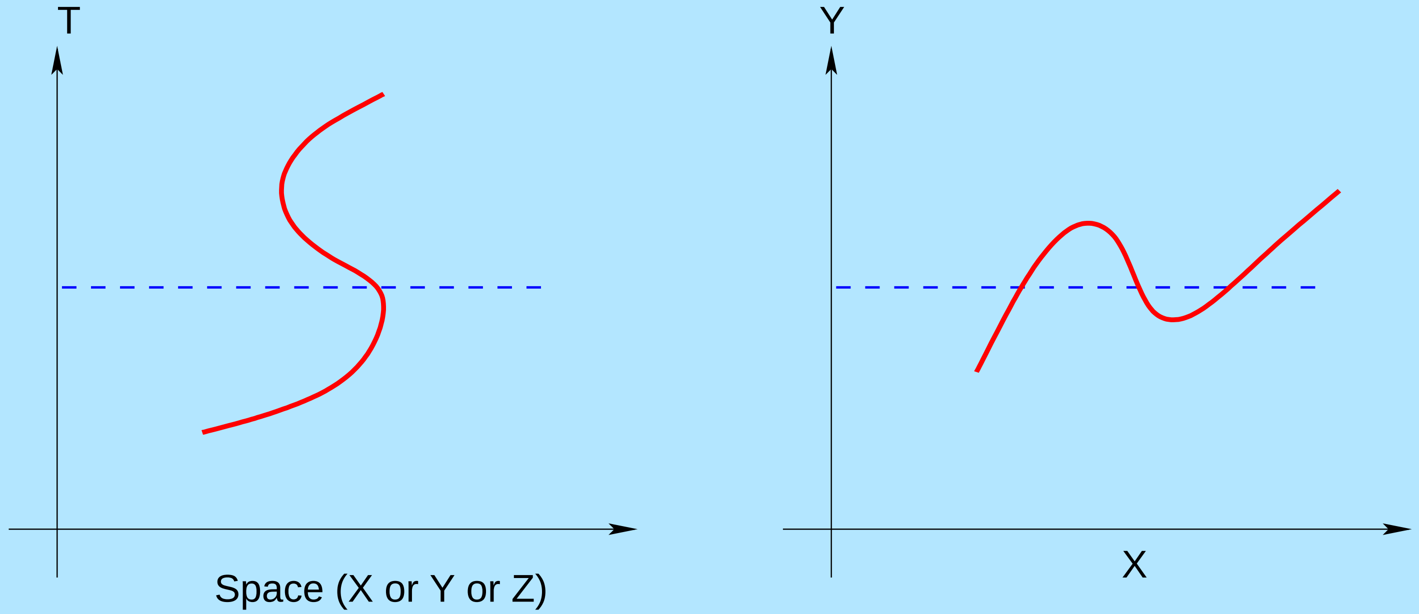
"I do not think we have a completely satisfactory relativistic quantum mechanical theory; even one, that does not agree with nature but at least agrees with logic. Therefore, I think the renormalisation theory is simply a way to sweep the difficulties of the divergences under the rug."

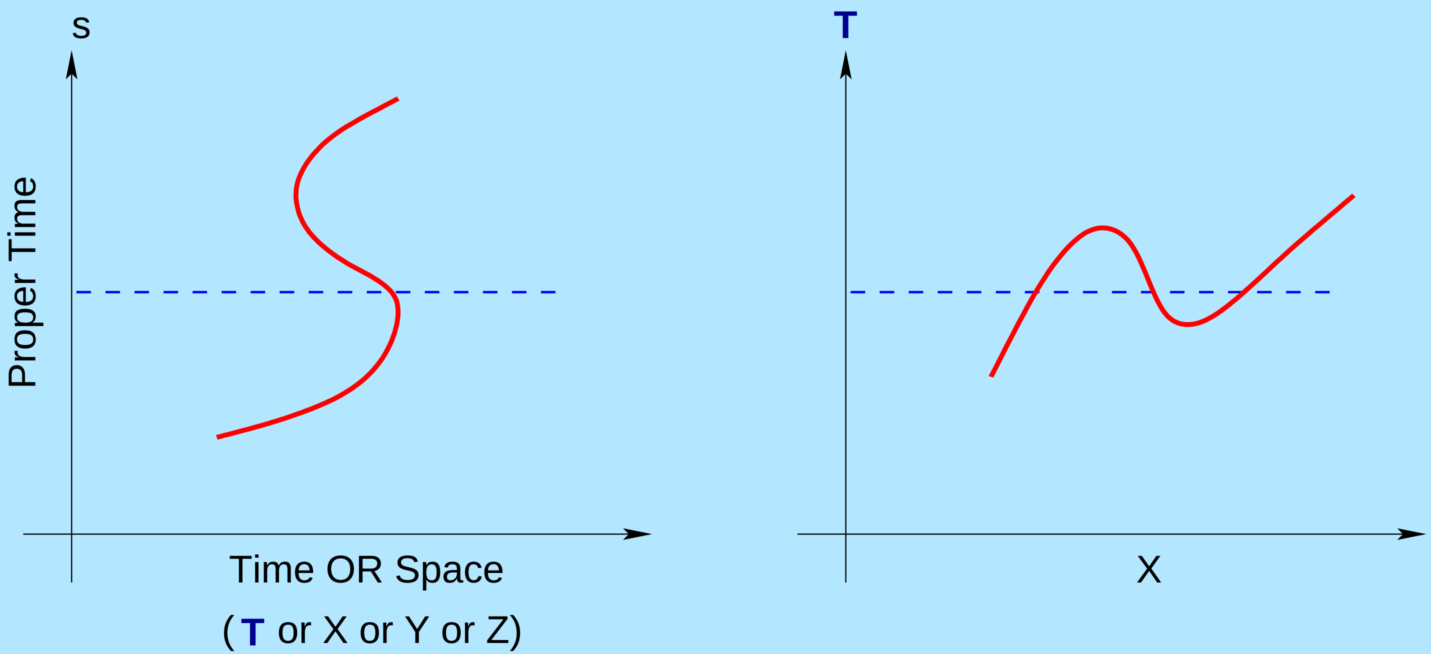
R.P.Feynman [1965, Nobel Prize lecture]

Non Relativistic Quantum Mechanics

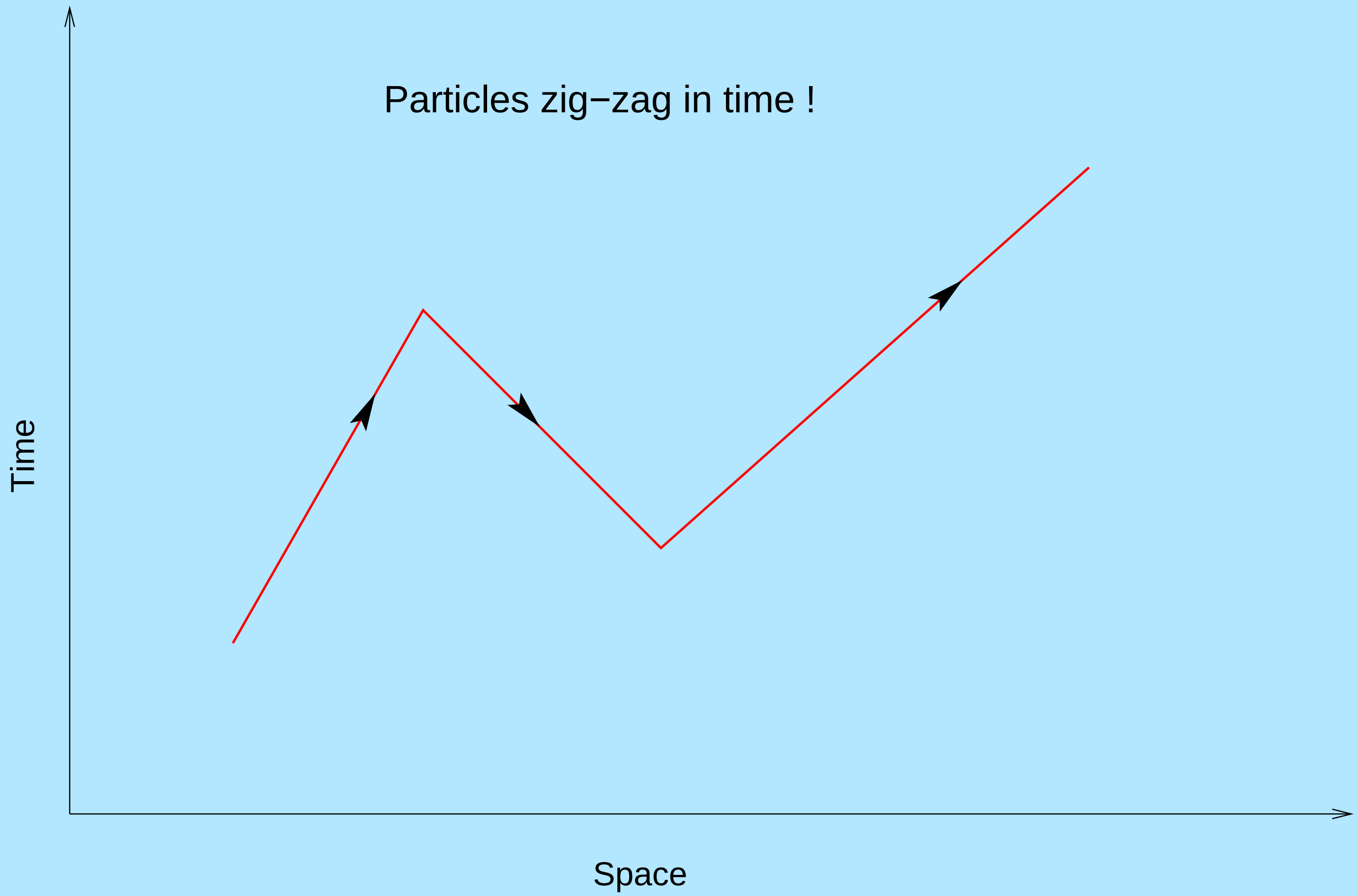


In NRQM a path is $[X(T), Y(T), Z(T)]$. Paths go backward in space X, Y, Z but not in time T

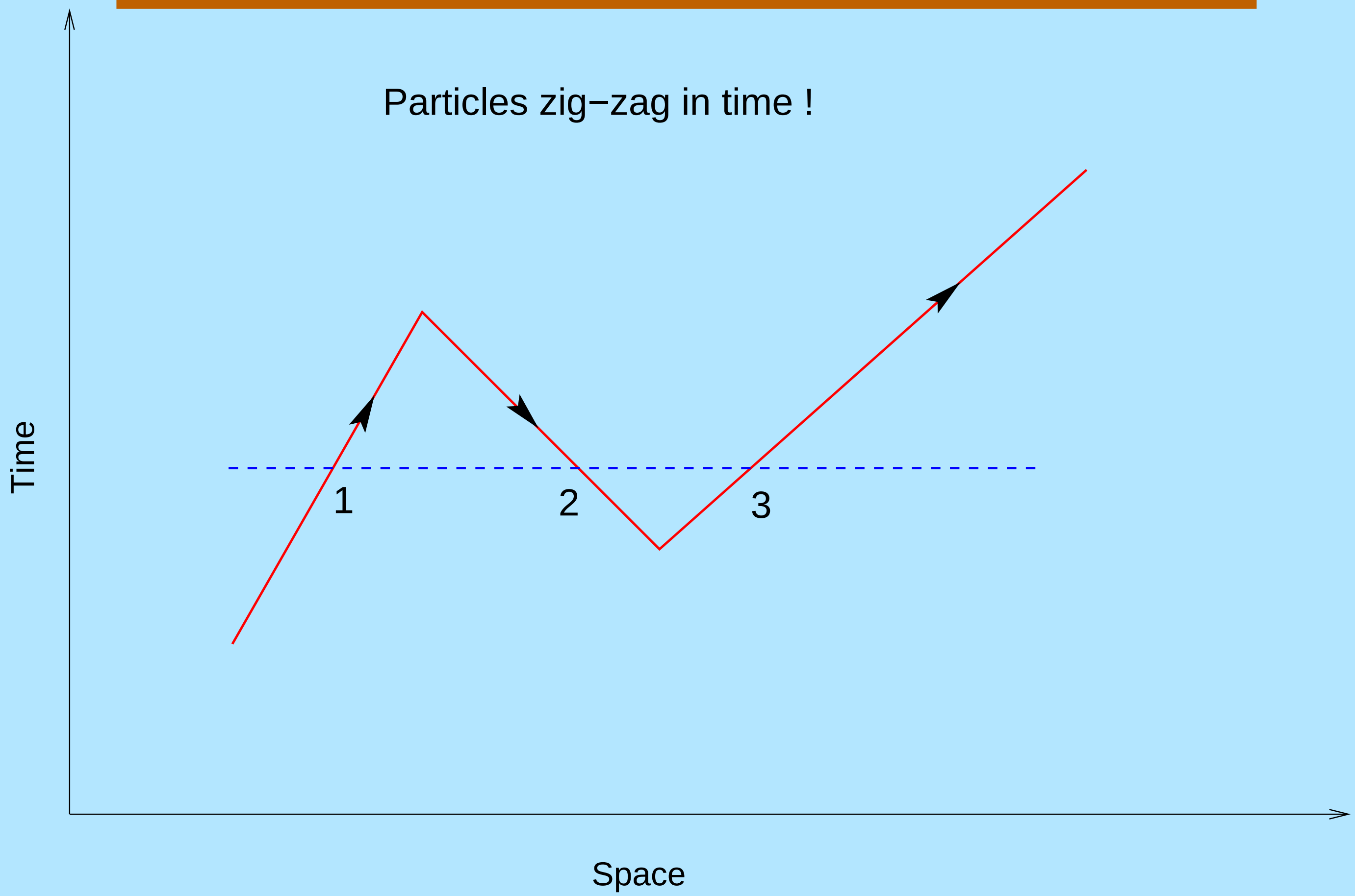
Relativistic Quantum Mechanics



In RQM a path is $[T(s), X(s), Y(s), Z(s)]$ where s is the proper time. Paths go backward in space X, Y, Z and time T !


Particles zig−zag in time !
Time
Space

Major Crisis : A particle is at 3 different places at a given time !?



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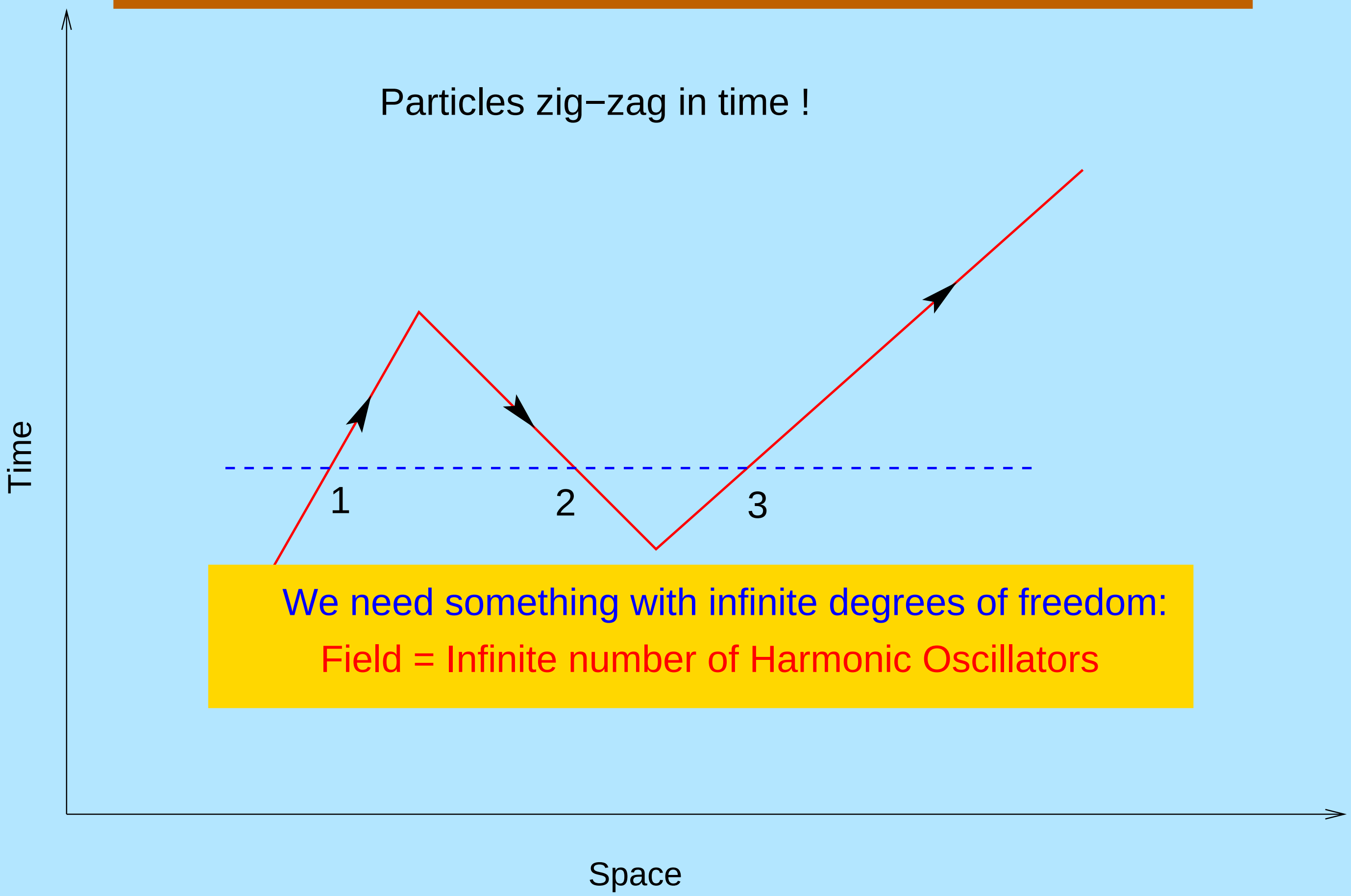
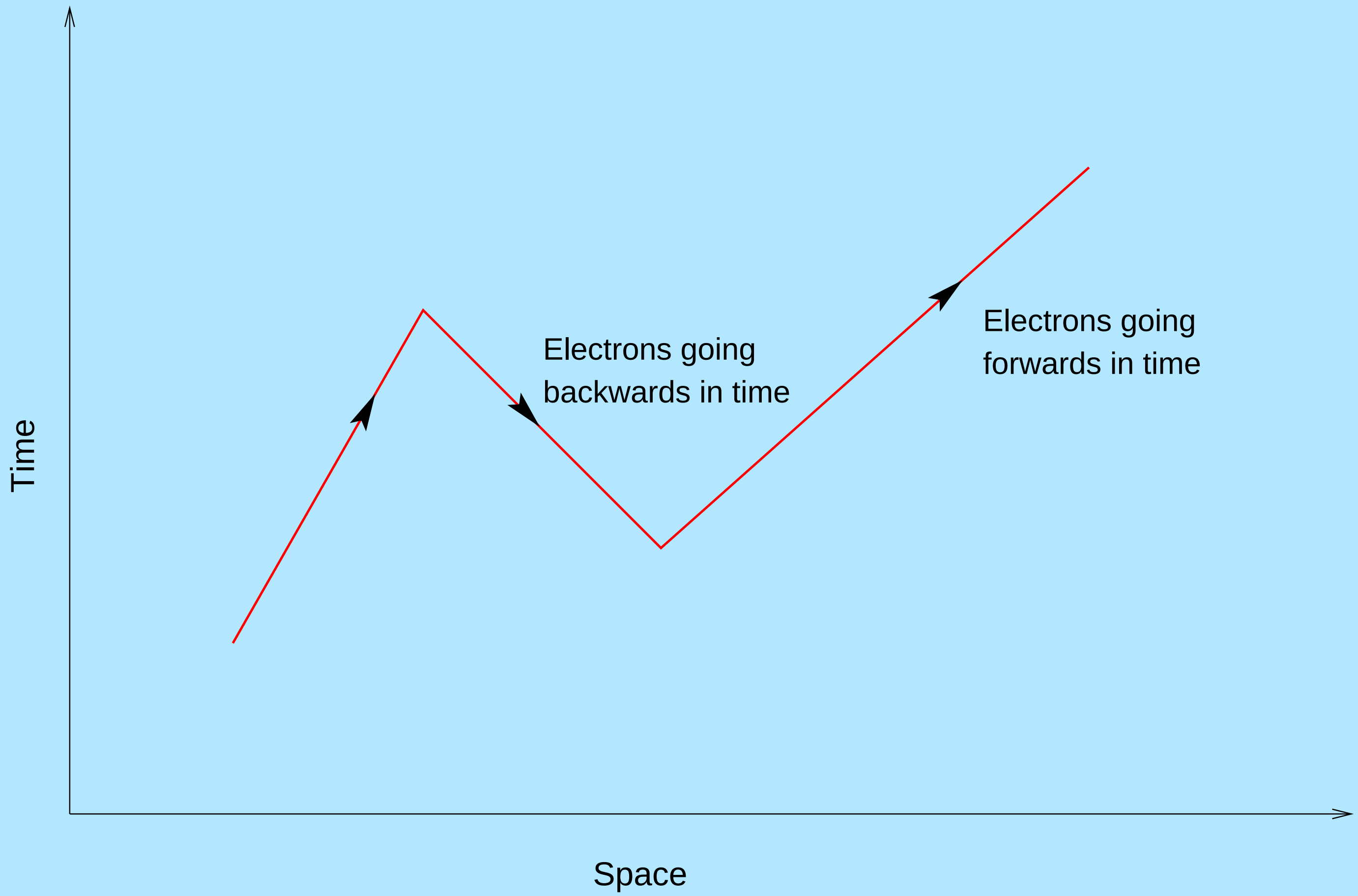
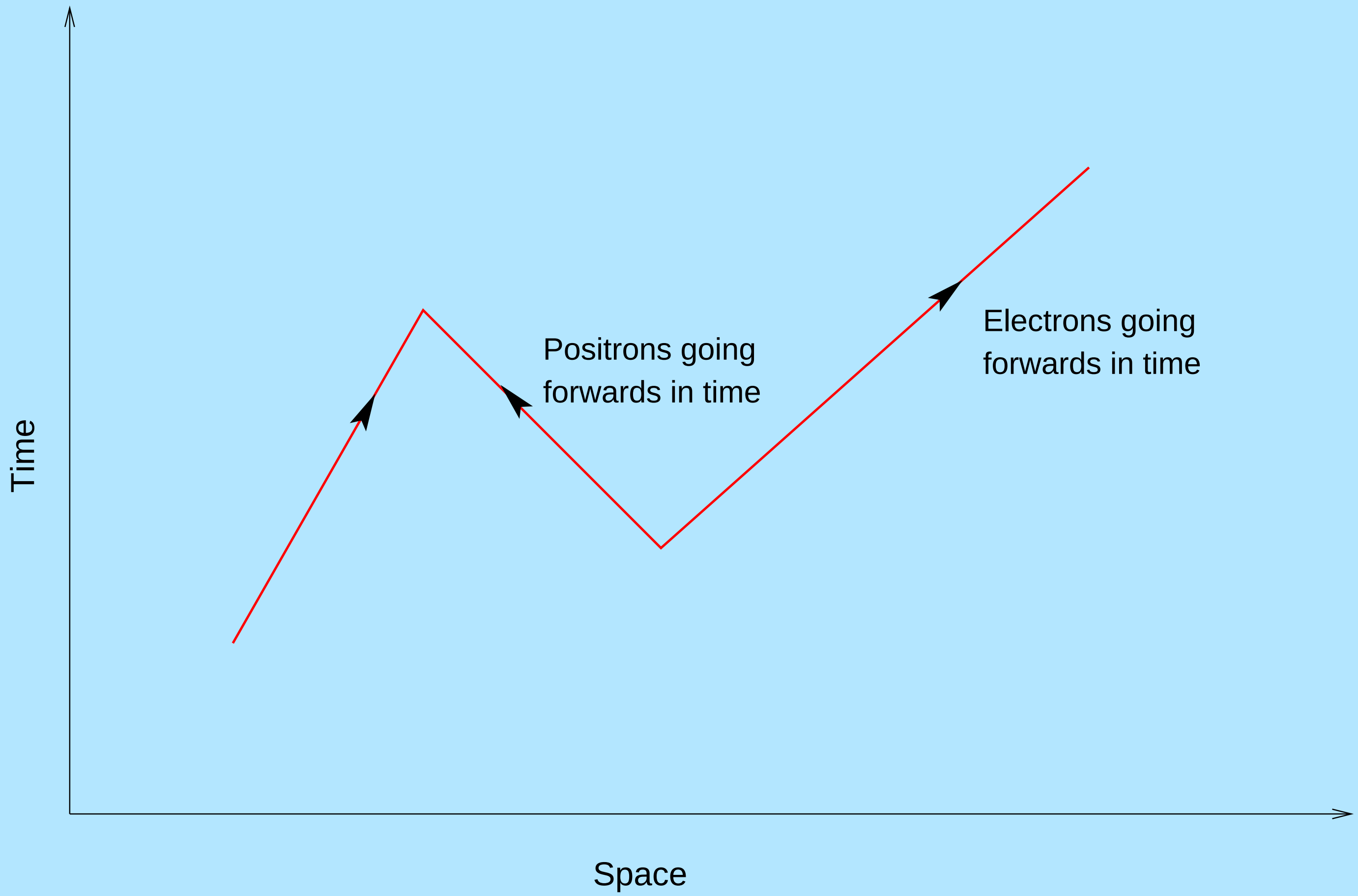
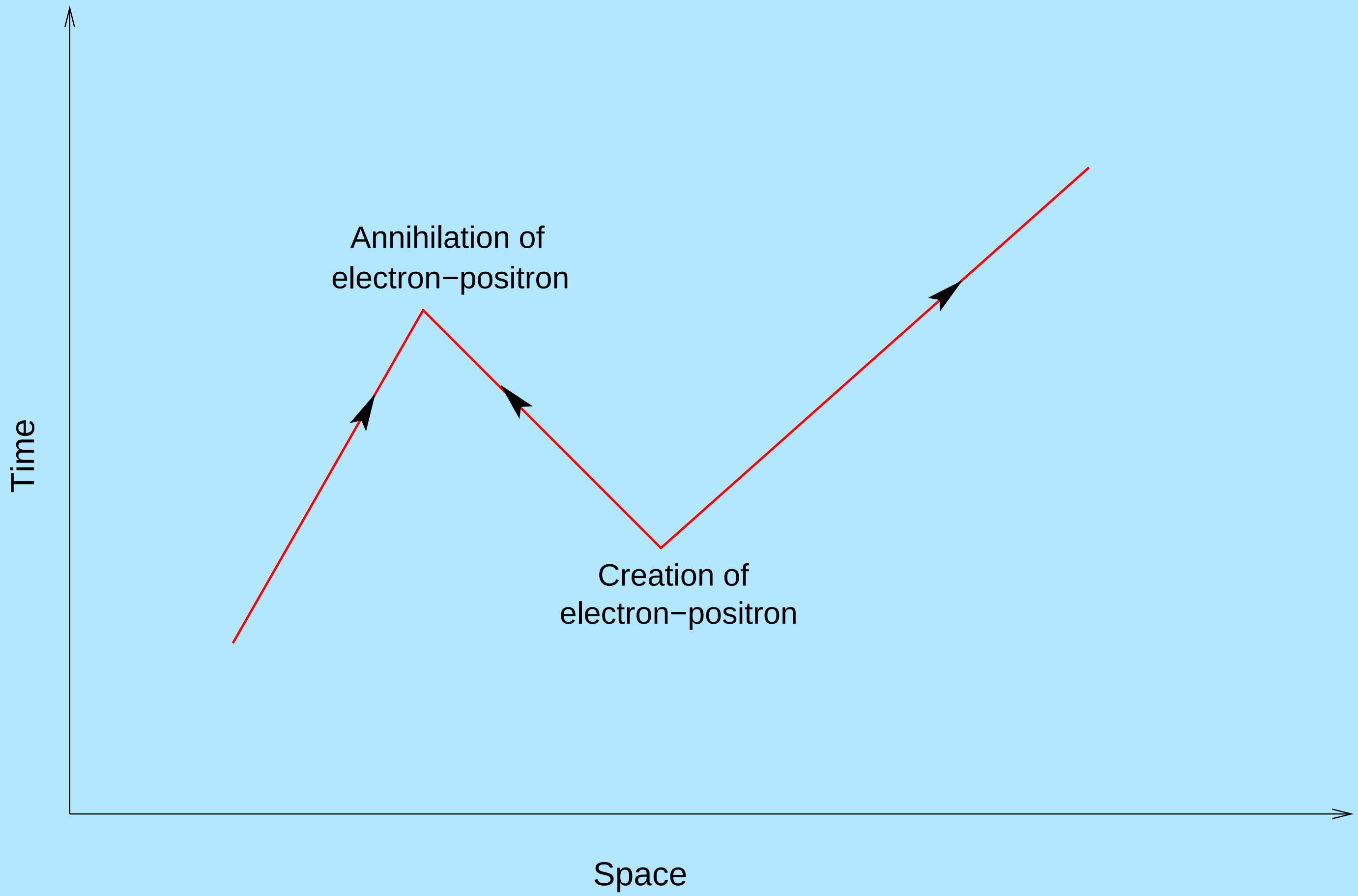


We need something with infinite degrees of freedom:

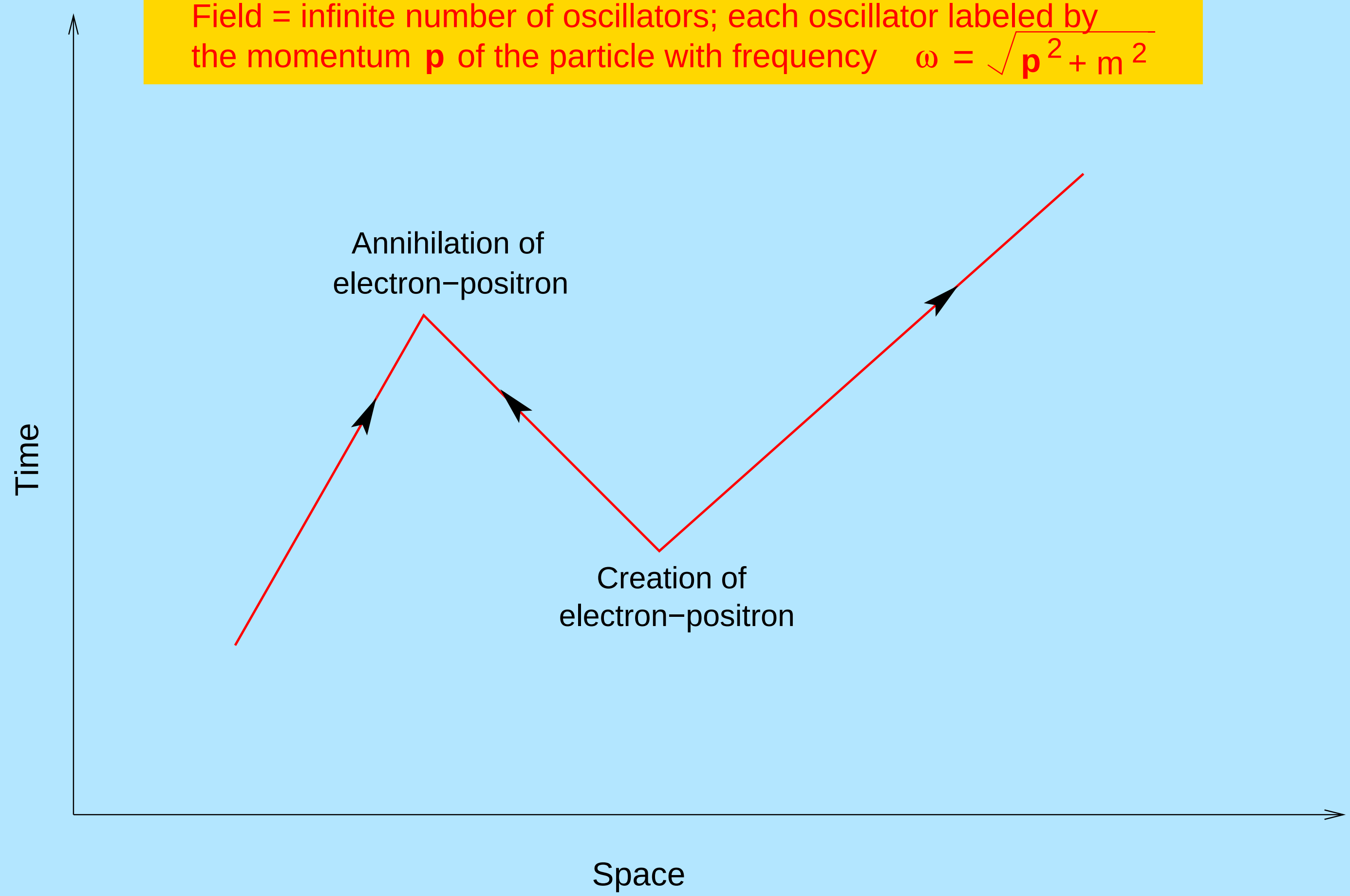
Field = Infinite number of Harmonic Oscillators

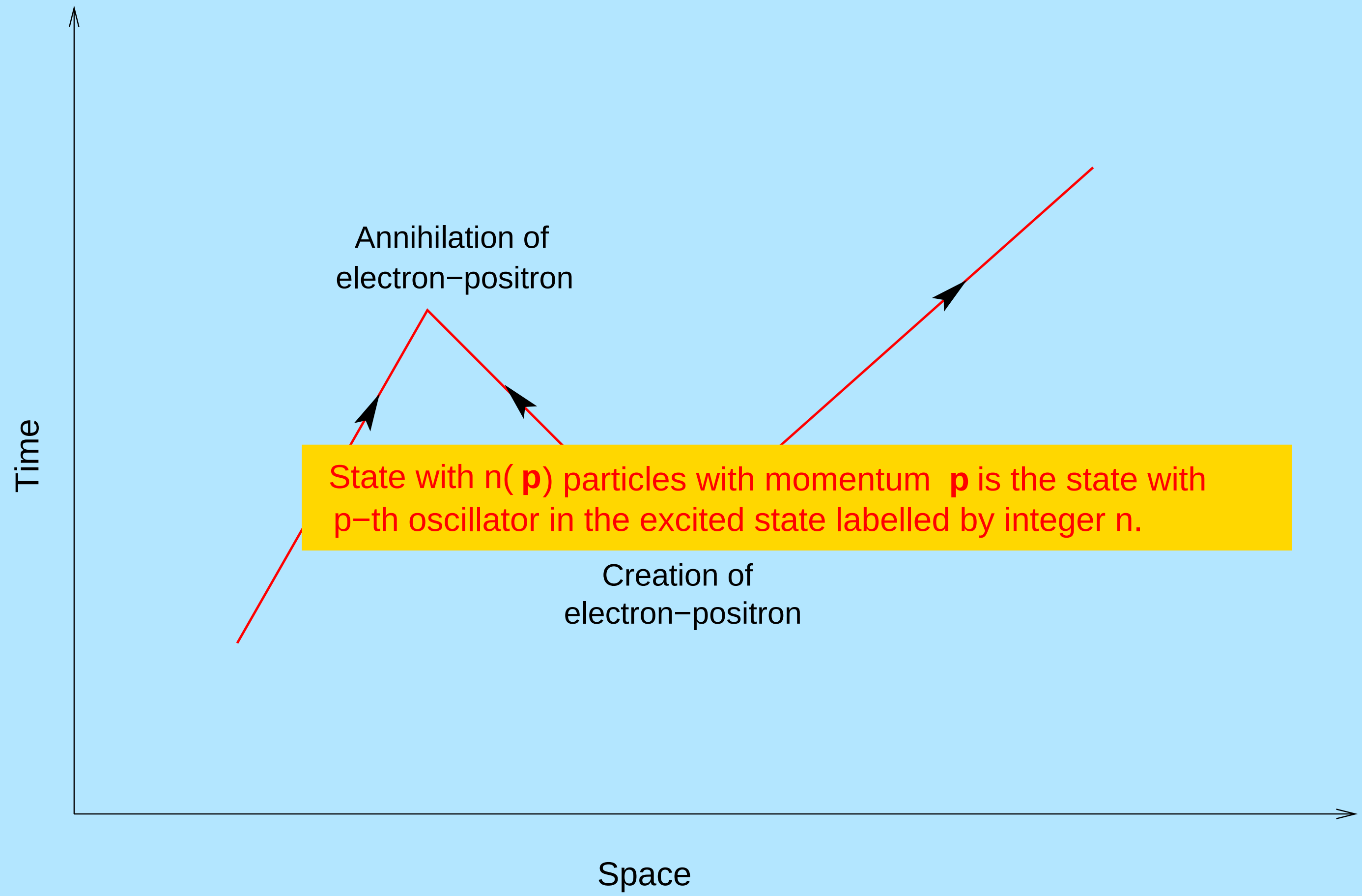

Time
Electrons going
backwards in time
Electrons going
forwards in time
Space


Positrons going
forwards in time
Electrons going
forwards in time
Time
Space

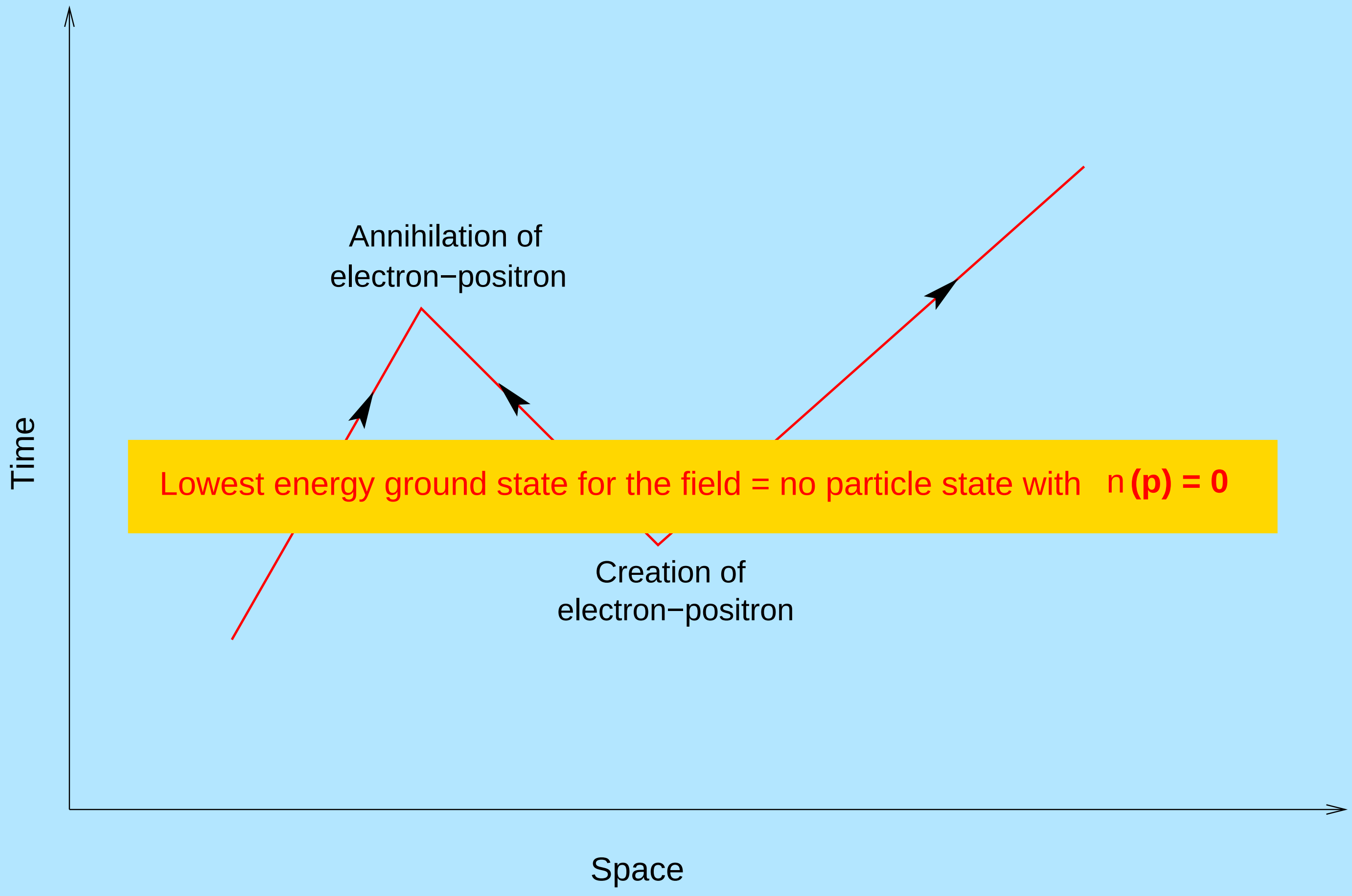

Annihilation of
electron−positron
Creation of
electron−positron
Time
Space

Field = infinite number of oscillators; each oscillator labeled by the momentum **p** of the particle with frequency $\omega = \sqrt{\mathbf{p}^2 + m^2}$



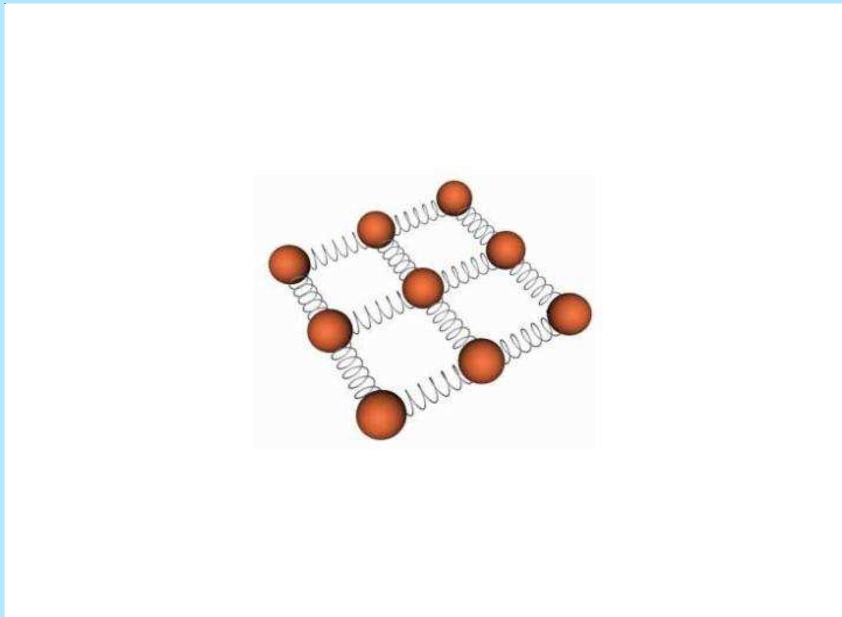


State with n(**p**) particles with momentum **p** is the state with p−th oscillator in the excited state labelled by integer n.


Annihilation of
electron−positron
Lowest energy ground state for the field = no particle state with n (p) = 0
Creation of
electron−positron
Time
Space

VACUUM DEPENDS ON THE SCALE OF PROBING

- Phonons, magnons, photons, electrons, protons, quarks ..... Each can arise as an excited state of a suitably defined system of oscillators.



- Example: Lattice vibrations when quantized will give rise to phonons. The ground state is that of zero phonons.

FIELD = INFINITE NUMBER OF OSCILLATORS

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- ..... is the same as a field

$$\phi(t, \mathbf{x}) = \int d^3\mathbf{k}\; q_{\mathbf{k}}(t)\; \exp i\mathbf{k}.\mathbf{x}$$

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$$\psi(q) = N \exp(-\frac{q^2}{2\Delta^2}); \qquad \Delta^2 =< q^2 >= \frac{\hbar}{2m\omega}$$

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- Ground state of electromagnetic field:

$$P \propto \exp\left[-\frac{1}{16\pi^3\hbar c}\int \frac{\mathbf{E}(\mathbf{x})\cdot\mathbf{E}(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^2} d^3x d^3y\right]$$

Key Point: You can never make the fluctuations go away.

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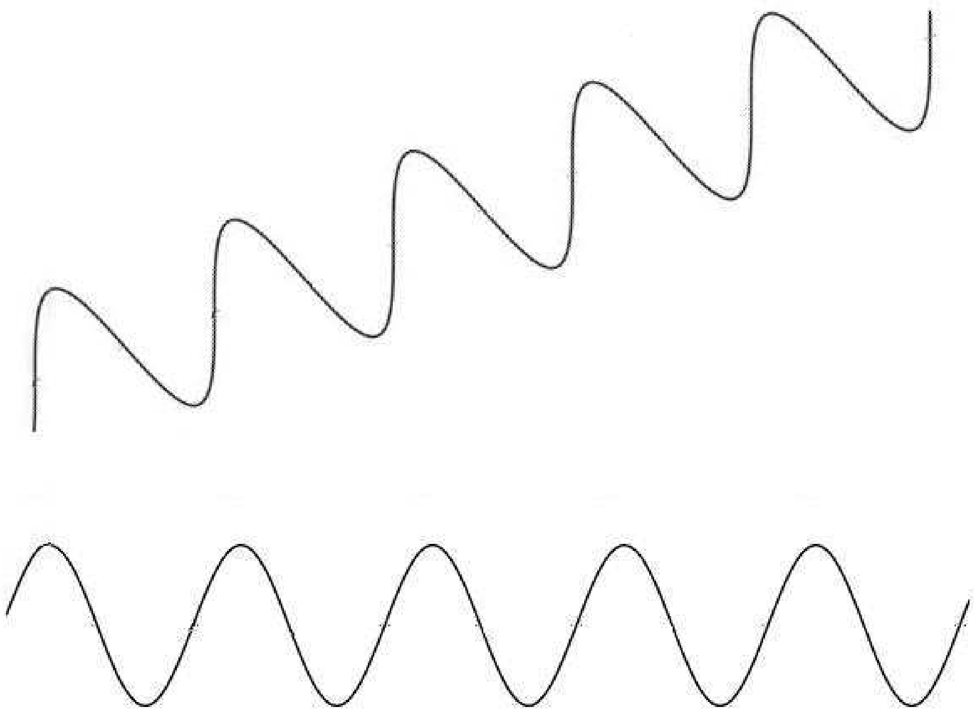
- Two conducting plates, kept in vacuum separated by a attracts each other with a force

$$\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 a^4}.$$

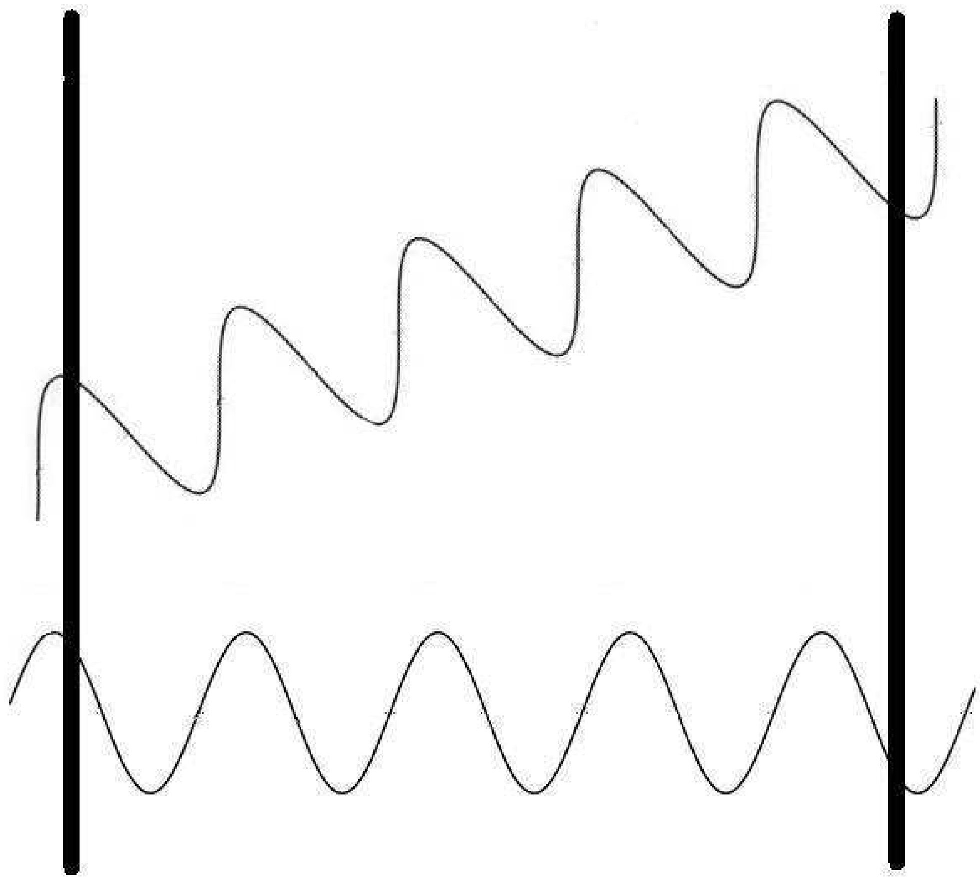
The energy of the configuration is

$$\frac{E}{A} = -\frac{\pi^2 \hbar c}{720 a^3}; \qquad F = -\frac{dE}{da}$$

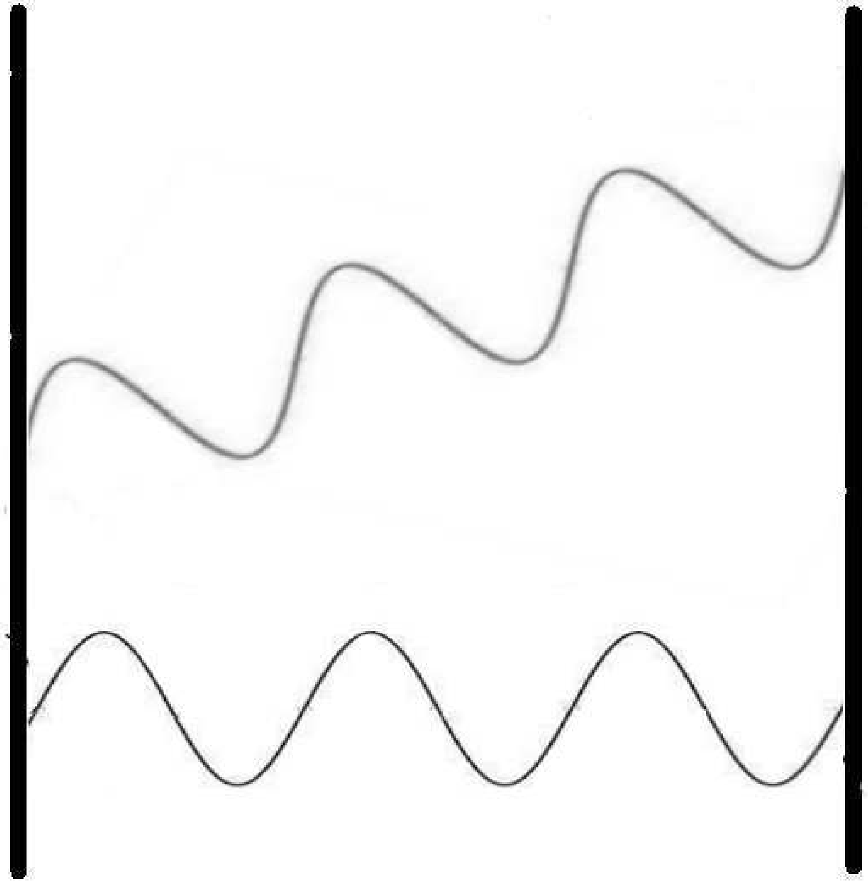
VACUUM STATE HAS RANDOM FIELD MODES



SOME OF THESE MODES VIOLATE BOUNDARY CONDITIONS AT PLATES



NEW VACUUM STATE IS DIFFERENT FROM THE ORIGINAL ONE!



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- One dimensional scalar field example:

$$\text{With plates}: k = (2\pi/a)n, n = 0, 1, 2, ...$$

$$\text{Without plates}: k = (2\pi/a)n, 0 < n < \infty$$

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- Difference in energy:

$$\Delta E = \frac{1}{2}\hbar \frac{2\pi c}{a}\left[\sum_n n - \int_0^\infty n dn\right] = -\frac{\pi \hbar c}{24 a}$$

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- Pattern of vacuum fluctuations change when external conditions change.

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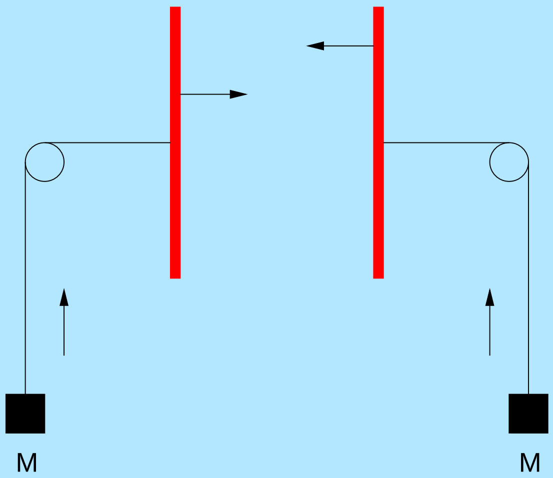

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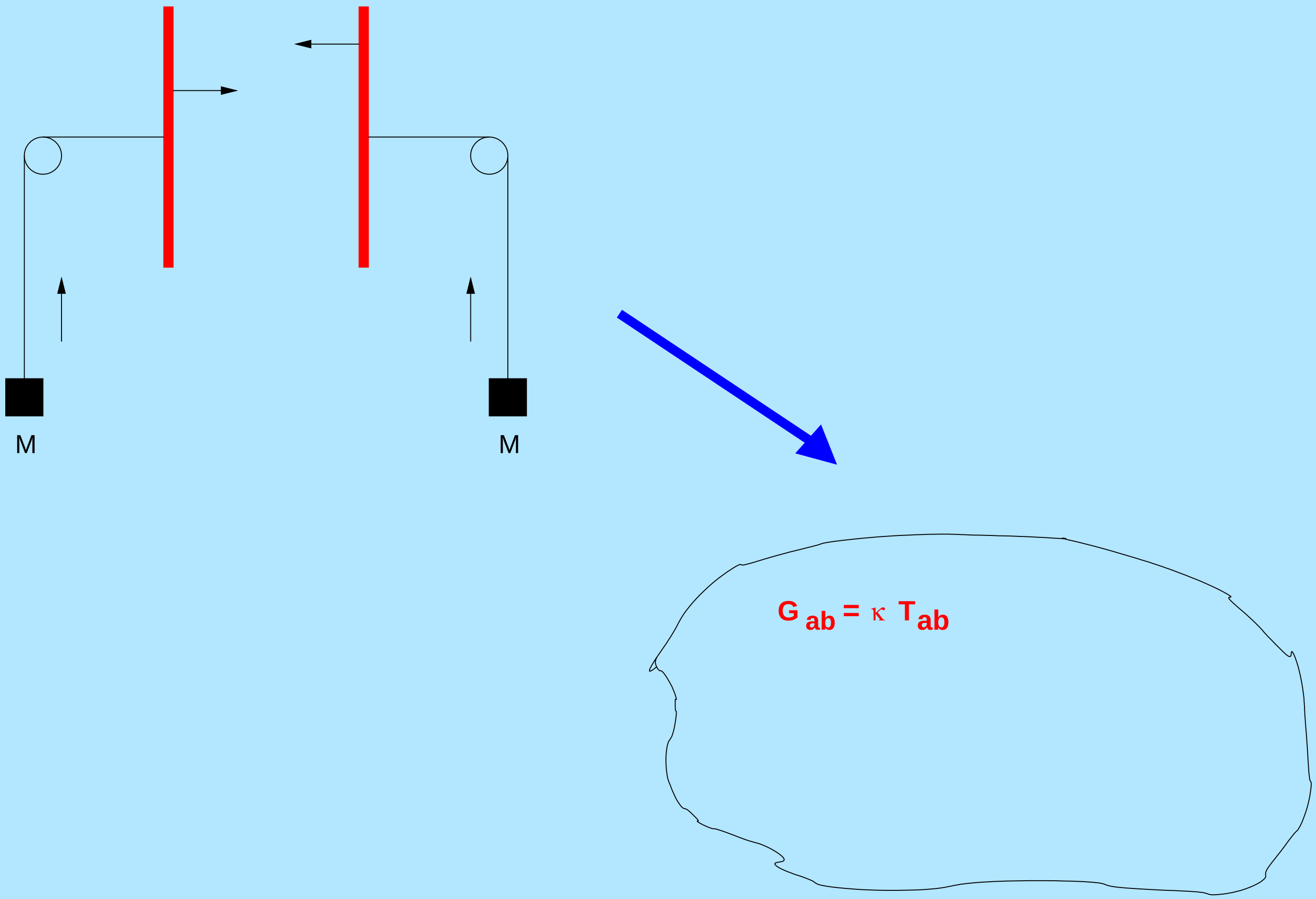

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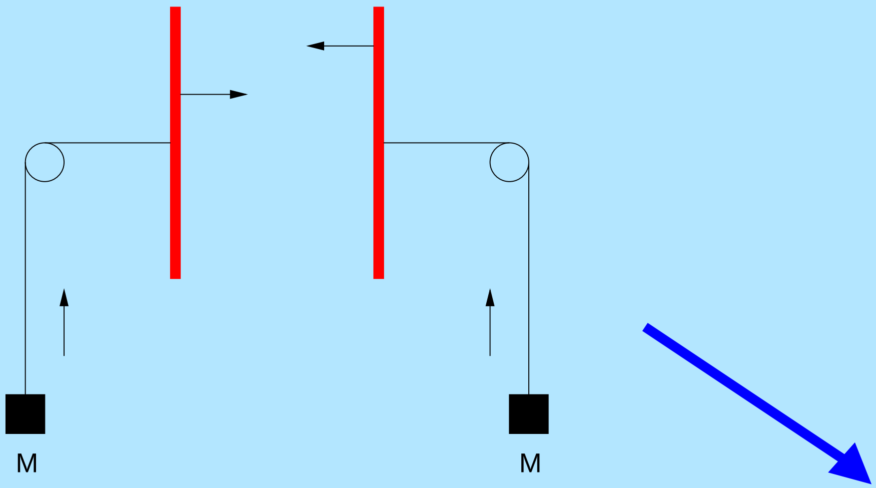

$$G_{ab} = \kappa \, T_{ab}$$

$$T_{ab} = T_{ab}(M)$$

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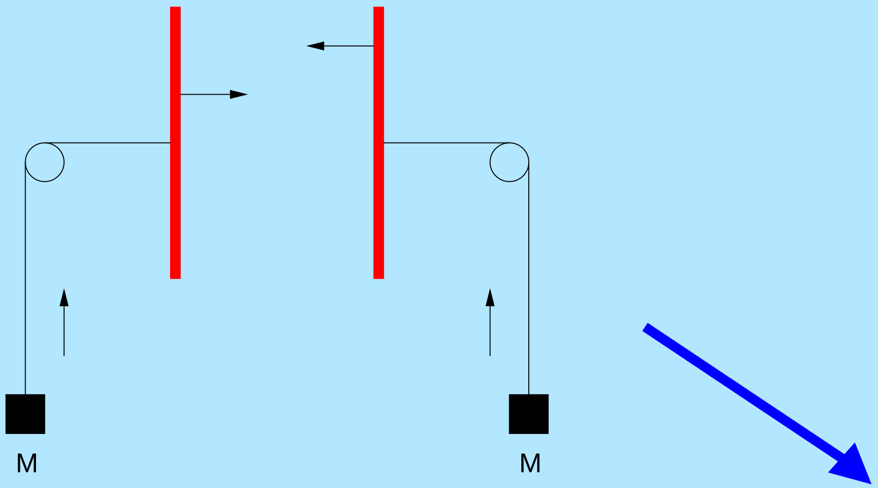


$$G_{ab} = \kappa \, T_{ab}$$

$$T_{ab} = T_{ab}(M) + T_{ab}(\text{vacuum})$$

IS VACUUM DOMINATING OUR UNIVERSE ?

- Our Universe has a totally preposterous composition which no cosmologist wanted !

- $\Omega_{\text{total}} \approx 1$

  $\Omega_{\text{radiation}} \approx 0.00005$

  $\Omega_{\text{neutrinos}} \approx 0.005$

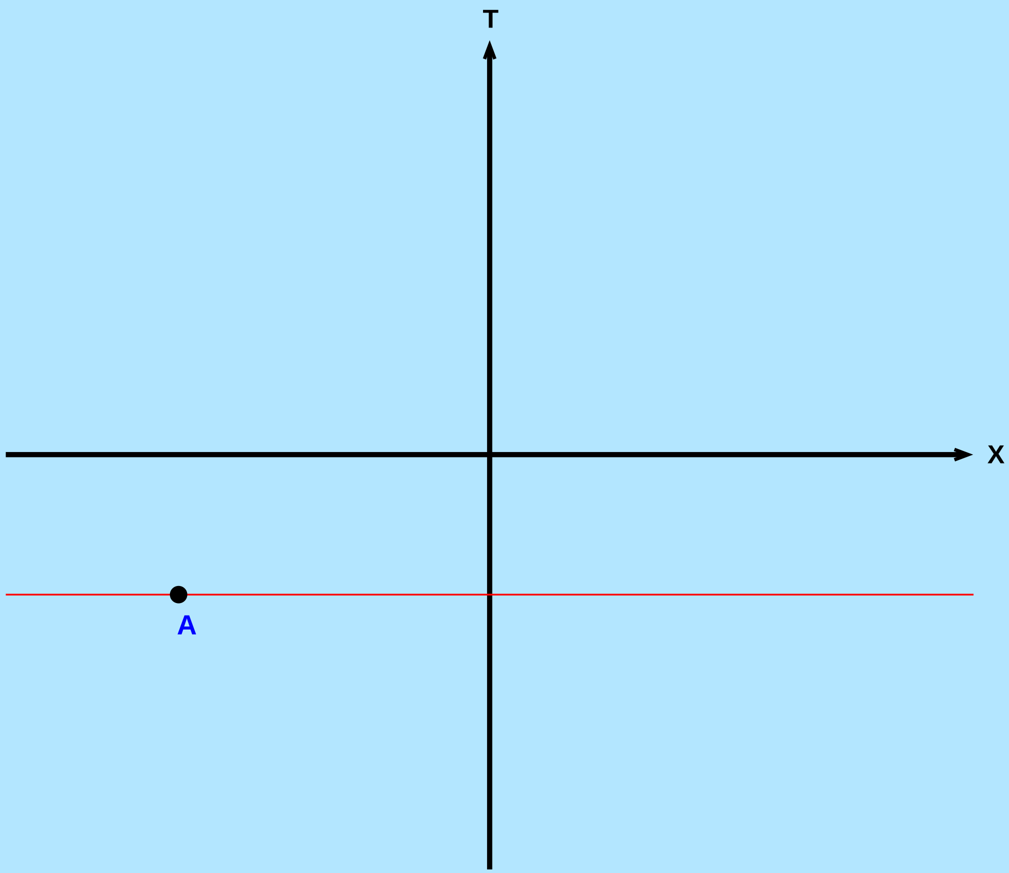
  $\Omega_{\text{baryons}} \approx 0.04$

  $\Omega_{\text{wimp}} \approx 0.31$

  $\Omega_{\text{darkenergy}} \approx 0.65$

- What is this Dark Energy ?

Spacetime and causal structure



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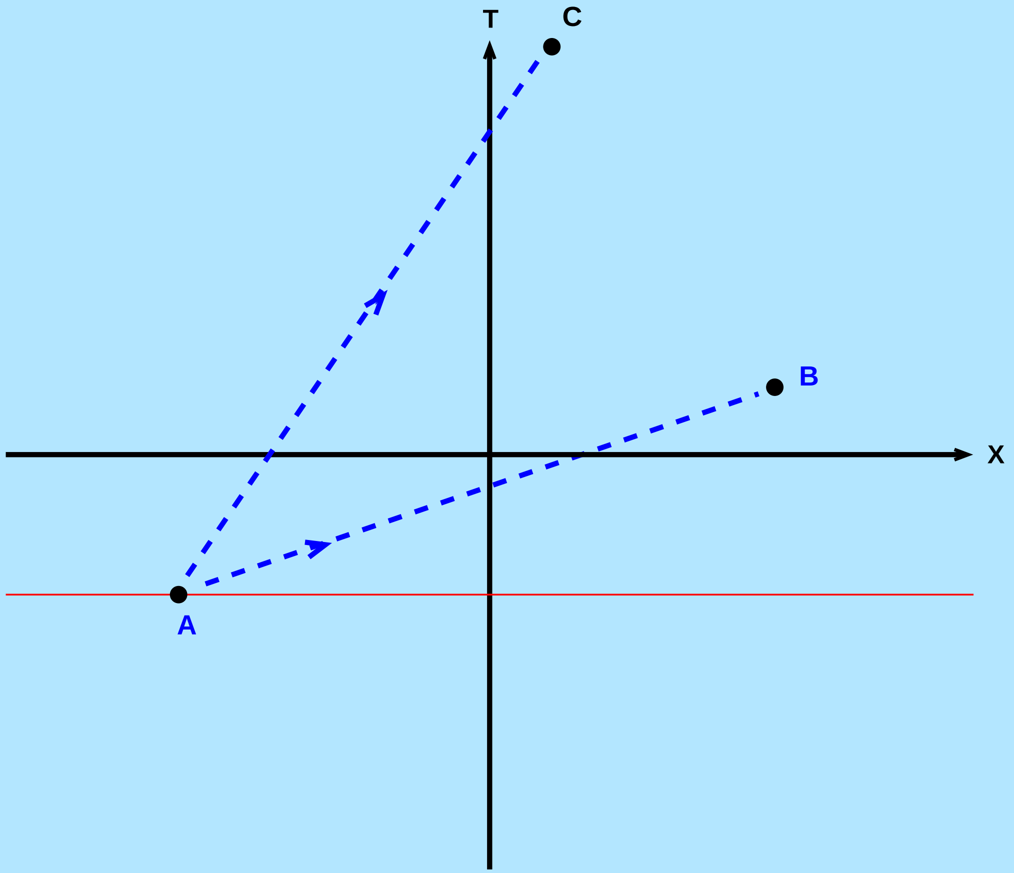
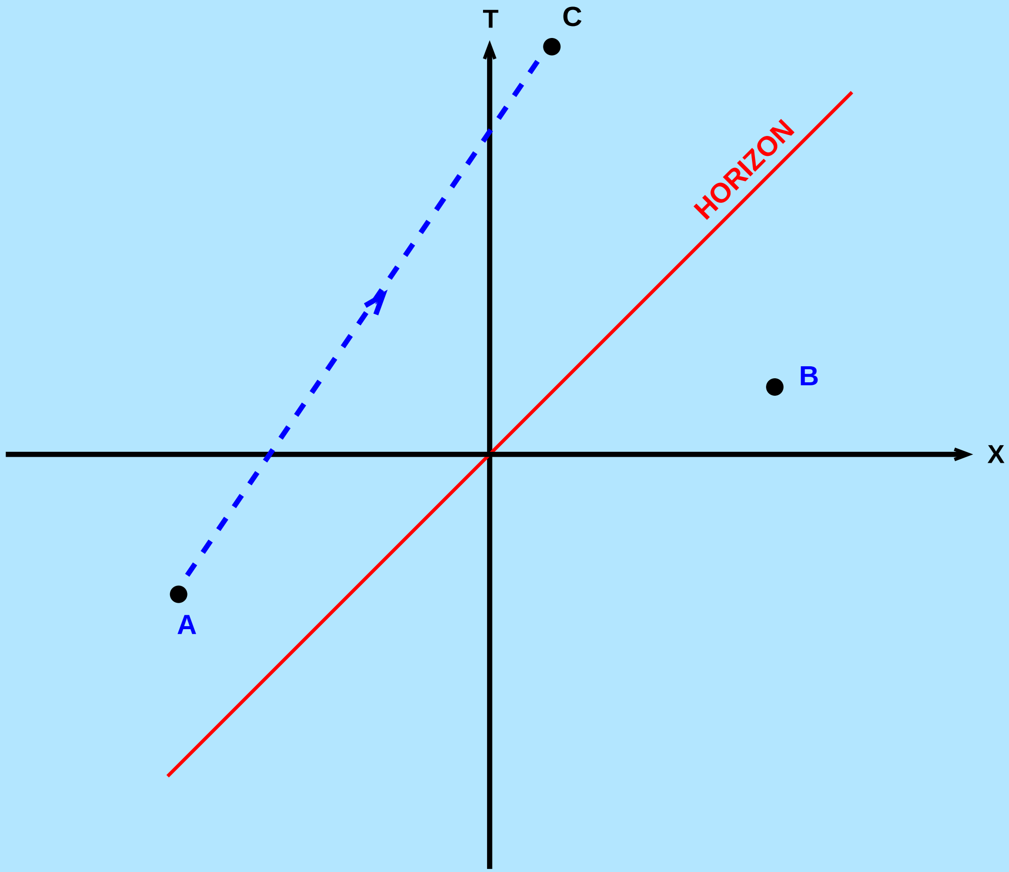


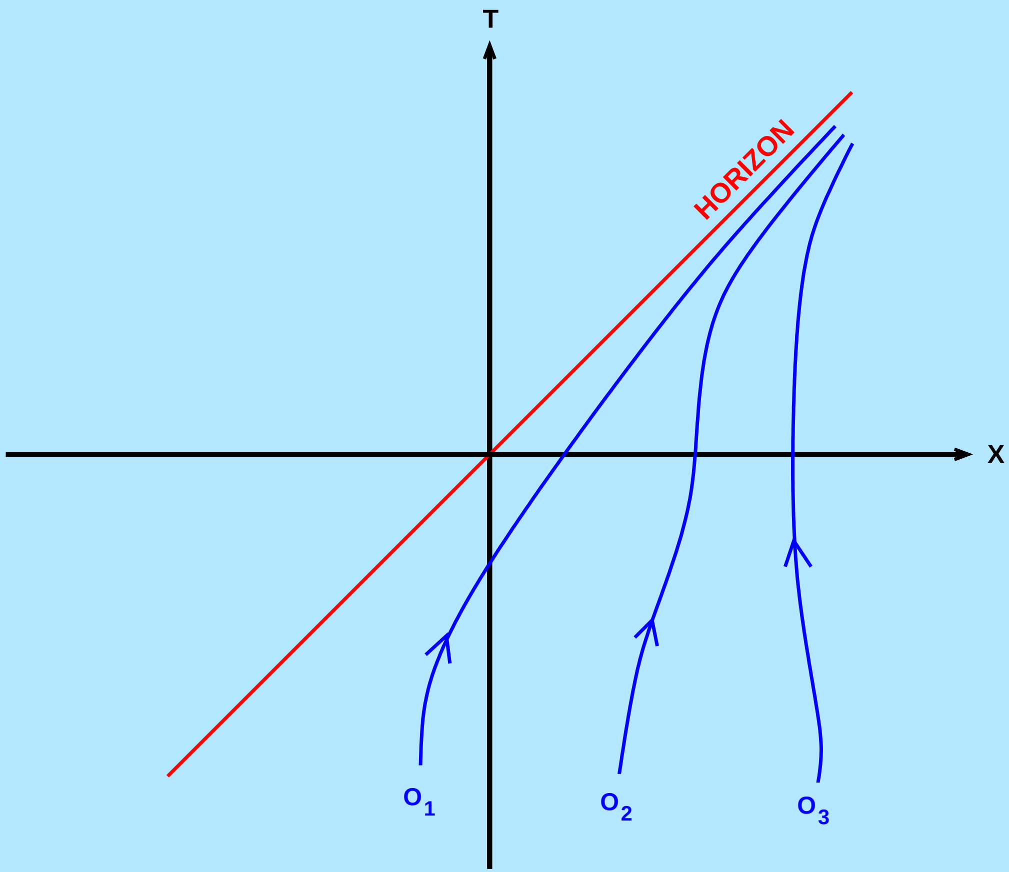
Newtonian physics: A can communicate with B and C.

Spacetime and causal structure



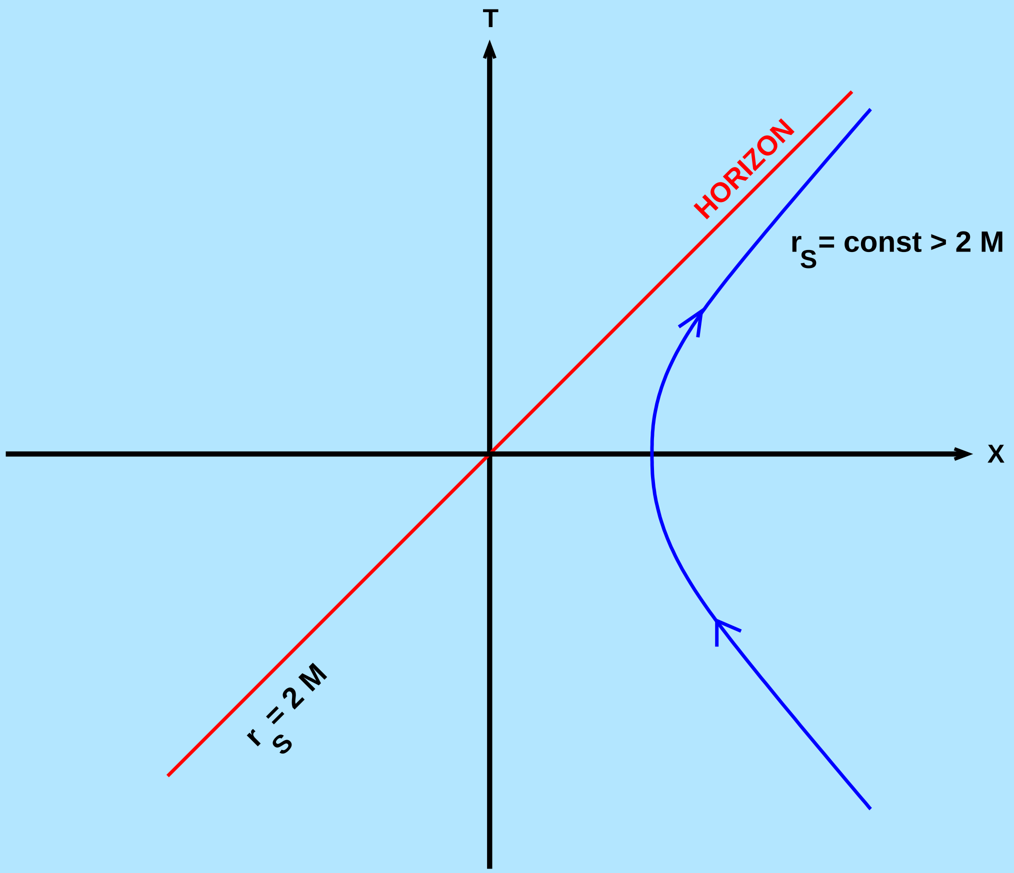
Special Relativity introduces a causal horizon; A cannot communicate with B.

Spacetime and causal structure



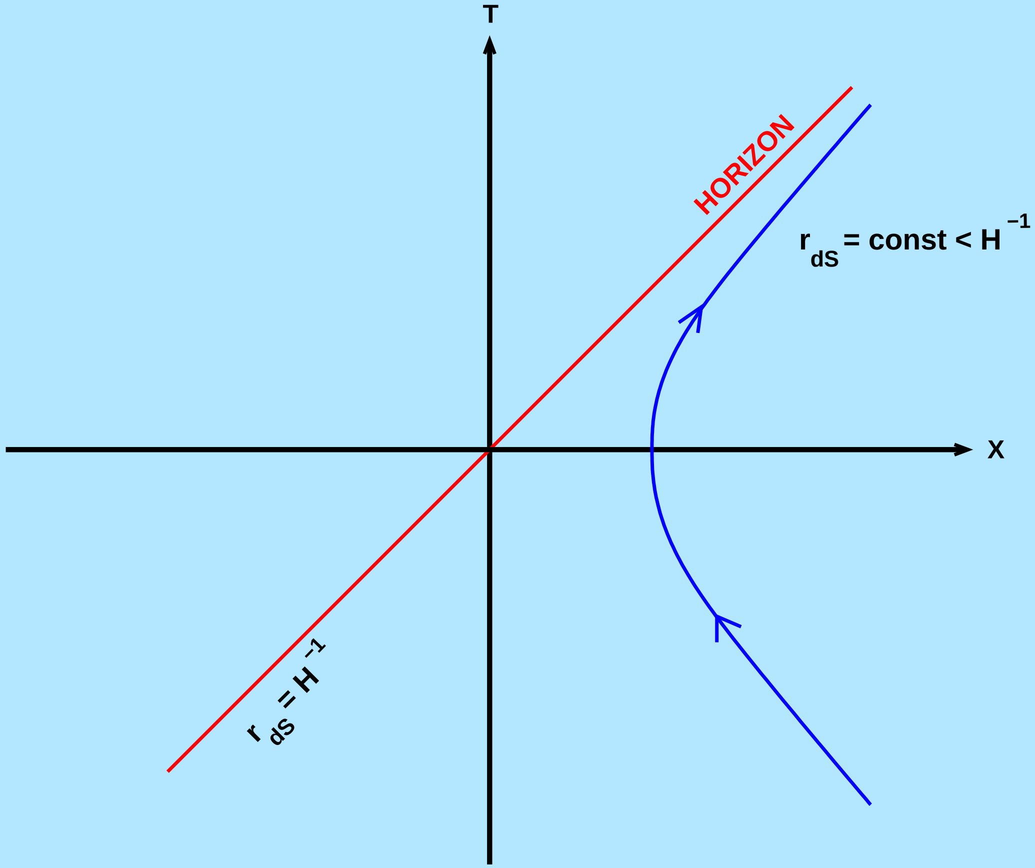
A family of observers ($O_1, O_2, O_3, \ldots$) have a causal horizon

Example 1: Schwarzschild spacetime



$$
\begin{aligned}
ds^2 &= -\left(1-\frac{2M}{r_{_S}}\right)dt_{_S}^2+\left(1-\frac{2M}{r_{_S}}\right)^{-1}dr_{_S}^2\\
&= Q_{_S}^2(T,X)(-dT^2+dX^2)
\end{aligned}
$$

Example 2: De Sitter spacetime



$$
\begin{aligned}
ds^2 &= -\left(1 - H^2 r_{dS}^2\right) dt_{dS}^2 + \left(1 - H r_{dS}^2\right)^{-1} dr_{dS}^2 \\
&= Q_{dS}^2(T, X)(-dT^2 + dX^2)
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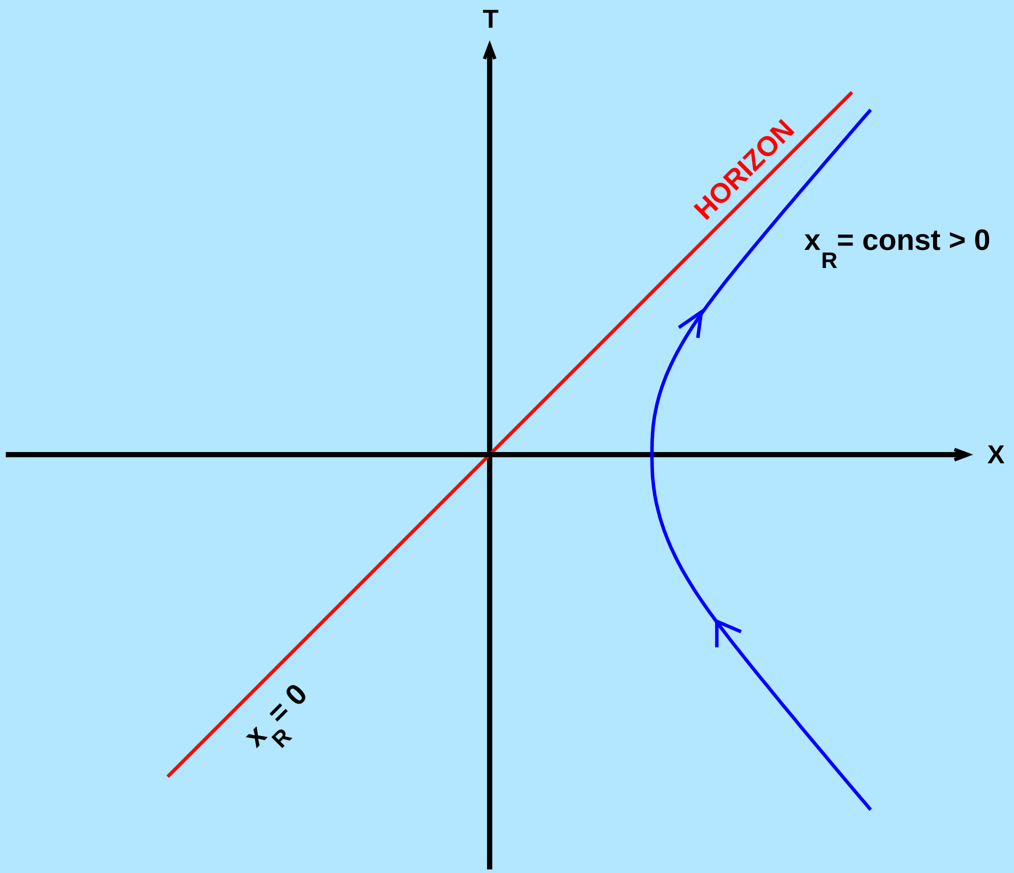
Example 3: Rindler (flat!) spacetime



$$ds^2 = -2\kappa x_R dt^2 + (2\kappa x_R)^{-1} dx_R^2$$
$$= -dT^2 + dX^2$$

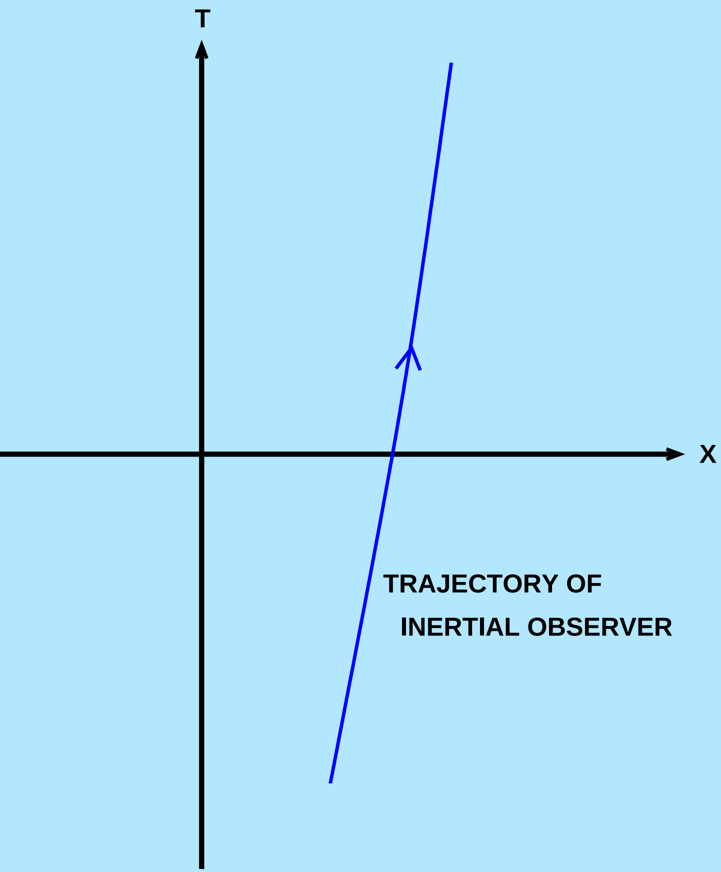
KEY NEW FEATURE
IN COMBINING GRAVITY AND QUANTUM THEORY

VACUUM STATE IS OBSERVER DEPENDENT!

Plane wave viewed by different observers
Warm-up: Inertial Observer

$$T = \frac{\tau}{(1 - v^2)^{1/2}}$$

$$X = \frac{v\tau}{(1 - v^2)^{1/2}}$$



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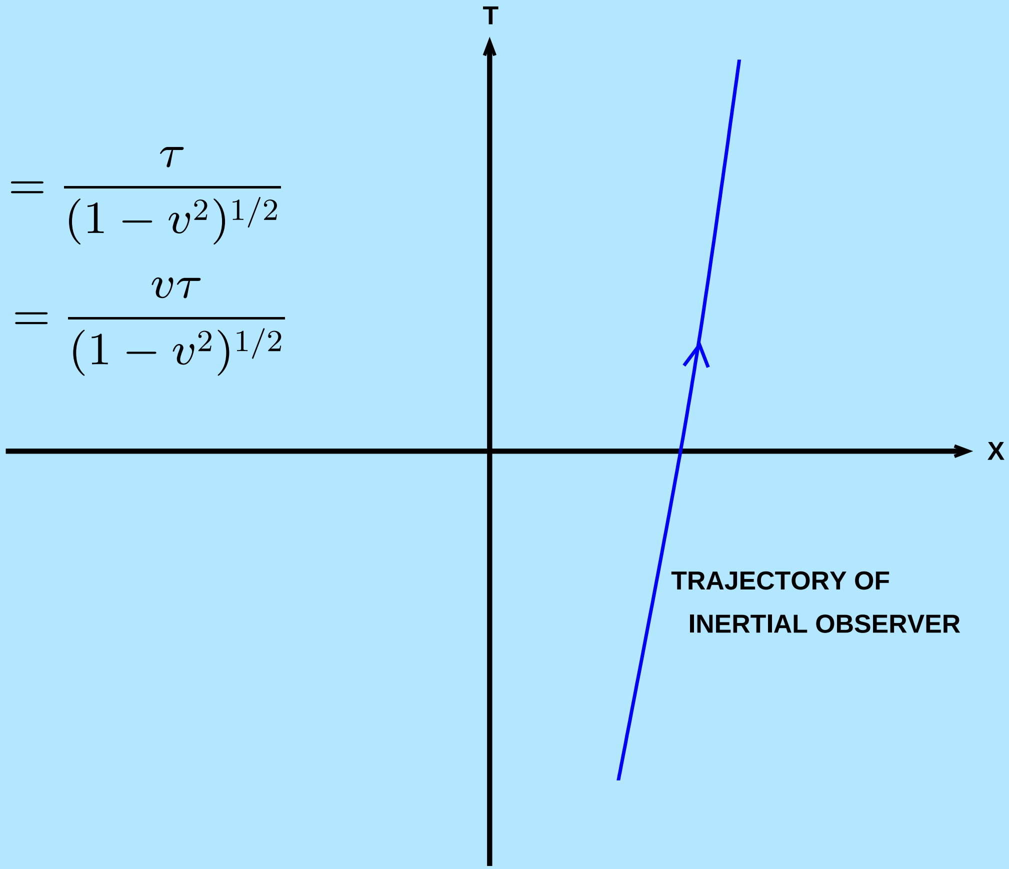

$$\phi(T,X) = \exp[-i\Omega(T-X)]$$

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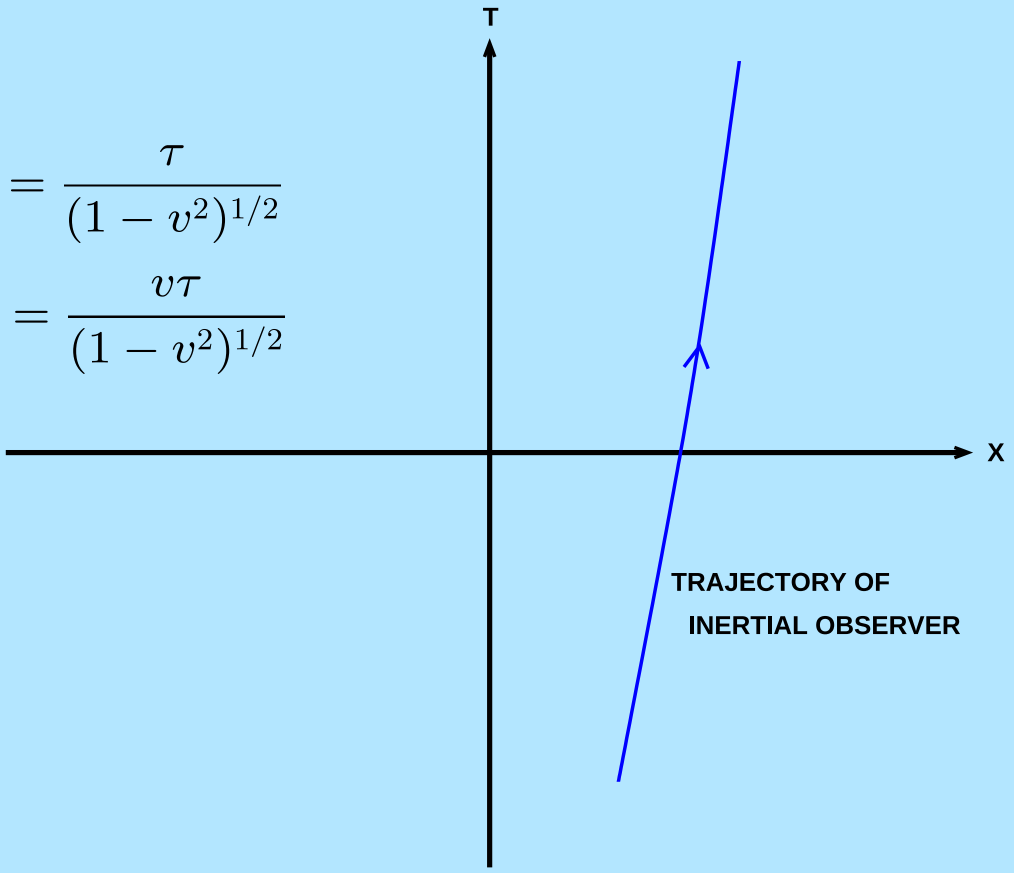


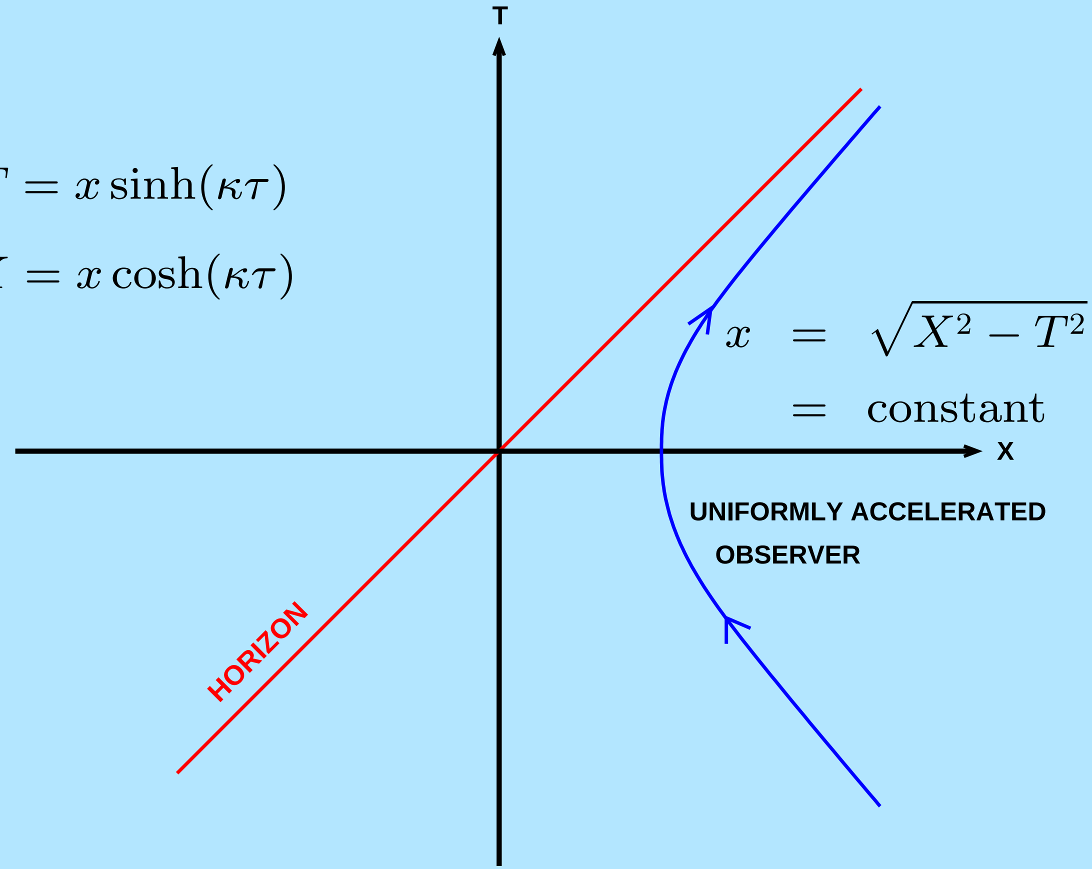
$$\phi(T,X) \equiv \phi\left(T(\tau), X(\tau)\right) = \exp -i\Omega \left(\frac{1-v}{1+v}\right)^{1/2} \tau$$

Doppler effect: $\Omega' = \Omega \left(\frac{1-v}{1+v}\right)^{1/2}$

Plane wave viewed by different observers
Observers with a causal horizon

$$T = x \sinh(\kappa\tau)$$

$$X = x \cosh(\kappa\tau)$$



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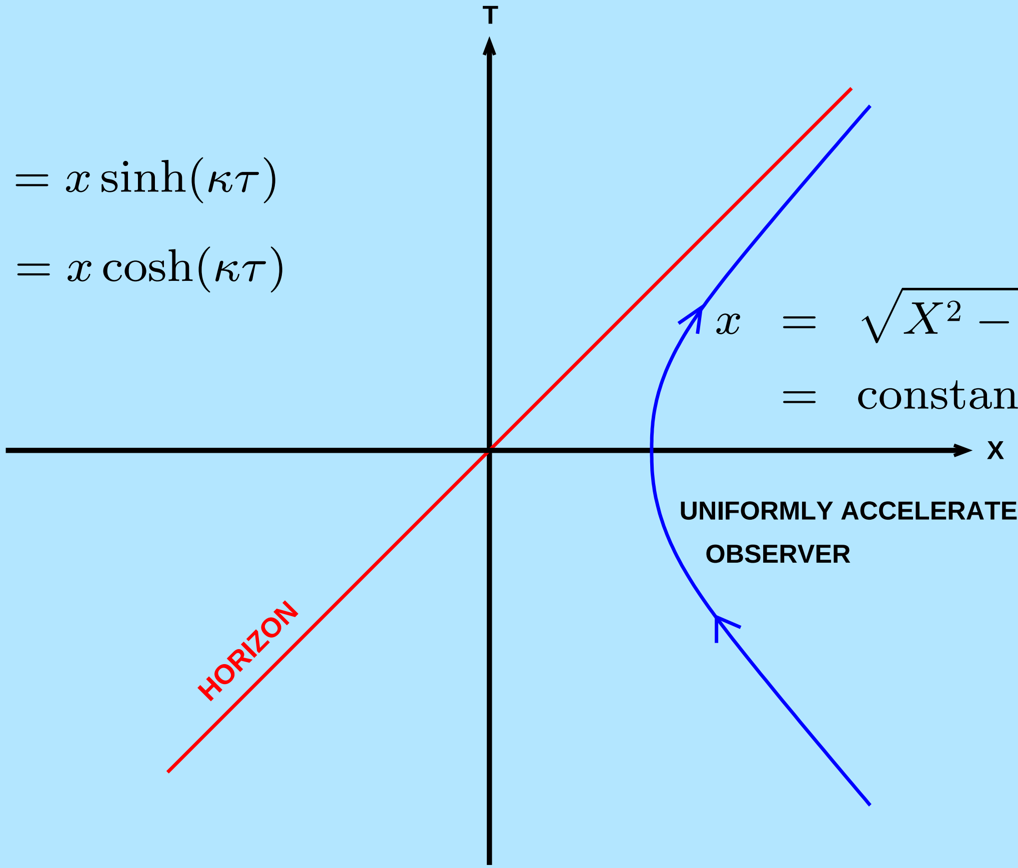

$$\begin{aligned} x &= \sqrt{X^2 - T^2} \\ &= \text{constant} \end{aligned}$$

$$\phi(\tau) = \phi(T(\tau), X(\tau)) = \exp i \overbrace{\left[\frac{\Omega}{\kappa} e^{-\kappa\tau} \right]}^{\text{exponential redshift!}}$$

RUNNING THROUGH THE VACUUM CAN HEAT YOU UP!

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- A mode $\phi(t,x) = \exp[-i\Omega(t-x)]$ frequency Ω will lead to

$$\phi(\tau) = \exp[i\Omega a^{-1}\exp(-a\tau)] = \int_0^\infty d\nu[A(\nu)e^{-i\nu t} + B(\nu)e^{i\nu t}]$$

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$$|A|^2 \propto \frac{1}{e^{\beta\nu}-1}; \qquad |B|^2 \propto \frac{e^{\beta\nu}}{e^{\beta\nu}-1}; \quad \beta = \frac{2\pi c}{a}$$

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- Exponential redshift occurs near any horizon with a determined by surface gravity at the horizon.

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$$|A|^2 \propto \frac{1}{e^{\beta\nu}-1}; \qquad |B|^2 \propto \frac{e^{\beta\nu}}{e^{\beta\nu}-1}; \quad \beta = \frac{2\pi c}{a}$$

- Exponential redshift occurs near any horizon with a determined by surface gravity at the horizon.

- For a black hole, $a = GM/R^2$ at $R = 2GM/c^2$. This gives $k_BT = \hbar c^3/8\pi GM$, the Hawking temperature.

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- In normal units, there is still no $\hbar$ in TdS!

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T.P.,gr-qc/0311036; gr-qc/0412068

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- We can obtain dynamics of gravity using *only* the surface term of the Hilbert action.

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Gravity as an Emergent Phenomenon

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of atoms/molecules

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KEY NEW RESULT: AREA QUANTISATION

In semi-classical limit, demanding $\exp iA_{sur} = \exp 2\pi i n$ leads to area quantization:

$$\text{Area} = (8\pi L_P^2)n$$

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- Third term: thermal energy density of de Sitter $\rho_{\text{rad}} \propto T_{\text{GH}}^4$

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- Obvious implications for quantum gravity.

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