Electrodynamics

Lecture 3.1
Units and Dimensions

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§ 1

The SI system of units

You already know about the mechanical units of kilogram, meter and second representing the dimensions of mass M, length L and time T. In terms of these units the unit of energy is a **joule**, unit of force is a **newton** and so on.

It is traditional in physics to name units after great physicists who contributed to knowledge closest to the area where that unit is employed. As unit names these are written without the capital letter, with no disrespect intended to the great men. And women. (What and how much is one curie?)

Electromagnetic fields require an additional dimension.

§ 2

Charge and Current

The natural choice for the additional dimension should be **charge** because the study of electromagnetic phenomena started with charge . However the fundamental electromagnetic quantity in the SI system is taken to be the electric **current**. Electric current i is defined as the flow of charge per unit time in a conducting wire. The unit of current is called an **ampere** equal to one **coulomb** per second where a coulomb is the unit of charge. The physical dimension of current will be denoted by [current]. (In general, the dimension of a quantity will be represented by that quantity enclosed by square brackets. We will be a little sloppy and use the same symbol for the units of the quantity as well.)

The unit of charge, **coulomb** has dimensions [charge] = [current]×[time].

It is a fact of nature, and it is something we do not fully understand, that the electric charge is always found to be a multiple of some fundamental natural unit of charge. This unit is taken by convention to be the charge of the electron. In this convention a coulomb is such that the electron charge is -1.6×10^{-19} coulombs.

(Add comments on fractional quark charges, Dirac charge quantization condition and running coupling constant in quantum electrodynamics.)

§ 3

Electric Field E and Displacement D

Electric field E is the force per unit charge and has dimensions

$$[\mathbf{E}] = \frac{[\text{force}]}{[\text{charge}]} = \frac{\text{newton}}{\text{ampere} \times \text{second}} \tag{1}$$

Another electrical unit is volt. A charge q gains energy $qE\,dx$ under the electric field of magnitude E when it moves a distance dx in the same direction as the field. The energy Edx gained per unit charge is called electric potential, and it is measured in a unit **volt**. Therefore, a useful unit for E is

$$[\mathbf{E}] = \frac{\text{volt}}{\text{meter}} \tag{2}$$

There is a quantity **D** which represents the influence or excitation that an electric field has on matter. Maxwell called it **dielectric displacement** and the name refers to the polarization or separation of positive and negative charges of neutral matter under the influence of **E**. The atomic nature of matter was still unclear in the second half of the nineteenth century. The free charges were supposed to cause the electric fields **E** which in turn caused the displacement **D**. The relation was given by the Gauss' law

$$\int_V \mathbf{D} \cdot \mathbf{n}\, da = \text{sum of free charges inside } V$$

This determines

$$[\mathbf{D}] = \frac{[\text{charge}]}{[\text{area}]} = \frac{\text{coulomb}}{\text{meter}^2} \tag{3}$$

This is a quantity very different from **E**. But it was found that the two are proportional to each other in most situations $\mathbf{D} = \epsilon\mathbf{E}$. When there is no matter but only free charges, the constant ϵ takes the value ϵ_0, called the **permittivity of empty space**

$$\mathbf{D} = \epsilon_0 \mathbf{E}. \tag{4}$$

It which has the dimensions

$$[\epsilon_0] = \frac{[\mathbf{D}]}{[\mathbf{E}]} = \frac{\text{coulomb}}{\text{meter}^2} \times \frac{\text{meter}}{\text{volt}} = \frac{\text{farad}}{\text{meter}} \tag{5}$$

Here we use the equation $Q = CV$ relating the voltage and charge of a capacitor and employ **farad** (equal to one coulomb per volt) as the unit of capacity. The numerical value of ϵ_0 is given by the relation

$$\epsilon_0 = \frac{10^7}{4\pi \times 9 \times 10^{16}} \approx 8.846 \times 10^{-12} \frac{\text{farad}}{\text{meter}} \tag{6}$$

§ 4

The Electromagnetic Constant c

There is another constant c of electromagnetic theory which has dimensions of velocity. Before the SI system of units was standardized, there were two principal systems - the so-called **electrostatic units** or esu and **electromagnetic units** or emu. In order to avoid a new dimension for electromagnetic quantities, it was decided to call an electrostatic unit of charge as that charge which repels by unit force an equal amount of charge at a unit distance by Coulomb's law.

$$[\text{force}] = \frac{[\text{charge}]^2}{[\text{distance}]^2} \tag{7}$$

This gives the dimensions of esu of charge in terms of mechanical dimensions

$$[\text{charge}]_{\text{esu}} = \sqrt{[\text{force}]}[\text{distance}] \tag{8}$$

On the other hand the electromagnetic definition depends on Ampere's law about attraction between current carrying wires. Recall that the force between two parallel 'current elements' at the ends of a line perpendicular to them is proportional to the product $(i_1 dl_1)(i_2 dl_2)$ and inversely proportional to square of distance between them. An emu of current is then the amount of current i which produces a force equal to $(dl)^2$ when equal and parallel current elements idl are kept a unit distance apart. Then

$$[\text{force}] = \frac{[\text{current}]^2_{\text{emu}}[\text{distance}]^2}{[\text{distance}]^2} = [\text{current}]^2_{\text{emu}} \tag{9}$$

Therefore,

$$[\text{current}]_{\text{emu}} = \sqrt{[\text{force}]} \tag{10}$$

An emu of charge then, is defined as the charge which passes through a wire with one emu of current in one second. It has the dimension

$$[\text{charge}]_{\text{emu}} = \sqrt{[\text{force}]}[\text{time}] \tag{11}$$

There is no reason for an esu of charge to match with an emu of charge. These quantities do not even have the same dimension! Actually, dimension-wise

$$\frac{[\text{charge}]_{\text{esu}}}{[\text{charge}]_{\text{emu}}} = [\text{velocity}] \tag{12}$$

which means that whenever charge is expressed in electrostatic units, we can convert it into electromagnetic units of charge by multiplying by a constant of the dimension of velocity.

This constant was called c and it was measured rather carefully by Weber and Kohlrausch in 1856. Their result was $c = 3.1 \times 10^{10}$ cm/sec. The extreme closeness of c to speed of light in vacuum made Kirchoff suggest a theory where the speed of propagation of waves of electric disturbances in a perfect conductor was equal to this constant c. Five years later in 1861 Maxwell gave his theory of propagation of electromagnetic waves, and it was recognized that light must be the same phenomenon. Electromagnetic waves were produced in the laboratory by Heinrich Hertz around 1888 and the measured speed agreed with the speed of light. Note, incidentally, that c is a large number (in usual units). Therefore **the emu of charge must be a very large unit compared to esu of charge**. Roughly speaking, magnetic effects are weaker by a factor of c compared to electric effects.

The the speed of light in vacuum in recent measurements is :

$$c = 2.997924590(8) \times 10^8 \frac{\text{meter}}{\text{second}} \tag{13}$$

§ 5

Magnetic Induction B and Magnetic Intensity H

Let us come to the units of magnetic type. Magnetic fields were discovered through natural magnets, and it was found to that they always occurred as magnetic moments or magnetic dipoles. The force between these magnetic dipoles was similar to the force between electric dipoles, and could be derived by assuming a similar inverse square law between poles. If we call by magnetic **pole-strength** the quantity analogous to the electric charge, then the natural quantity to define is a magnetic field (similar to the electric field)

$$[\mathbf{B}] = \text{“the magnetic field”} = \frac{[\text{force}]}{[\text{pole strength}]} \qquad (14)$$

Unfortunately, due to historical confusions and careless nomenclature, the currently used word for **B**, the rightful owner of the name magnetic field, is **magnetic induction**.

Adding insult to injury, there is a totally different quantity **H** which is given the name **magnetic field intensity**. We shall come to that shortly.

It was discovered by Ampere that a small closed wire loop of area A and with current i behaves exactly as if it was a magnet of dipole magnitude iA. This fact can be used to relate the quantity pole-strength to current :

$$[iA] = \text{ampere} \times \text{meter}^2 = [\text{pole strength}] \times \text{meter} \qquad (15)$$

or

$$[\text{pole strength}] = \text{ampere} \times \text{meter} \qquad (16)$$

Thus the unit of **B** turns out to be

$$[\mathbf{B}] = \frac{\text{newton}}{\text{ampere} \times \text{meter}} \qquad (17)$$

Notice the relation between dimensions of **E** and **B** :

$$[\mathbf{B}] = \frac{[\text{force}]}{[\text{charge/time}][\text{length}]} = \frac{[\mathbf{E}]}{[\text{velocity}]} \tag{18}$$

The unit of magnetic induction is **tesla**. Sometimes an older, smaller, unit called **gauss** is also used. One tesla is equal to 10^4 gauss.

The electric displacement **D** includes a term the effects of electric polarization which is measured by electric dipole moment per unit volume with the dimensions [charge][length]/[volume] or [charge]/[area].

Similarly, there is a quantity **H** called the the **magnetic field intensity**. **H** represents the magnetic excitation in a material due to magnetic induction **B**. It includes magnetic polarization which has the dimensions of magnetic moment per unit volume or (what is the same thing), [pole-strength]/[area] in complete analogy to the electric case. Thus,

$$[\mathbf{H}] = \frac{[\text{pole-strength}]}{[\text{area}]} = \frac{\text{ampere}}{\text{meter}} \tag{19}$$

Just as **D** and **E** are related linearly, so are **H** and **B** are related linearly for fields not too strong. But the proportionality constant μ is written *not* as $\mathbf{H} = \mu\mathbf{B}$ as one would expect because **H** is dependent on **B** but as $\mathbf{B} = \mu\mathbf{H}$ ("which is illogical" grumbled the great physicist Arnold Sommerfeld.)

Be that as it may, the constant μ when there is no magnetic matter around, is written μ_0 which is called the **magnetic permeability** of empty space. It has the dimensions

$$[\mu_0] = \frac{[\mathbf{B}]}{[\mathbf{H}]} = \frac{\text{newton}}{\text{ampere}^2} \tag{20}$$

"There is one more horrible thing.", R. P. Feynman says. When magnetic materials *are* present, (and μ is different form

μ_0) the quantity $\mu_0\mathbf{H}$ which has the same dimensions as $\mathbf{B}$ is measured in terms of a unit called 'oersted'. One oesrted is exactly equal to one gauss, they have the same dimension, but sadly, conventions have no logic. Luckily, the use of oersted is limited only to old books.

You can check that the dimensions of μ_0 and ϵ_0 are related by a constant of the dimensions of a square of velocity

$$[\mu_0] = \frac{1}{[\epsilon_0][\text{velocity}]^2}.$$

But there are no prizes for guessing what this velocity constant is. As a matter of fact,

$$\epsilon_0\mu_0 = \frac{1}{c^2} \tag{21}$$